RobotSoft 2025

Clarke Transform and Encoder-Decoder Architecture for Arbitrary Joint Locations in Displacement-Actuated Continuum Robots

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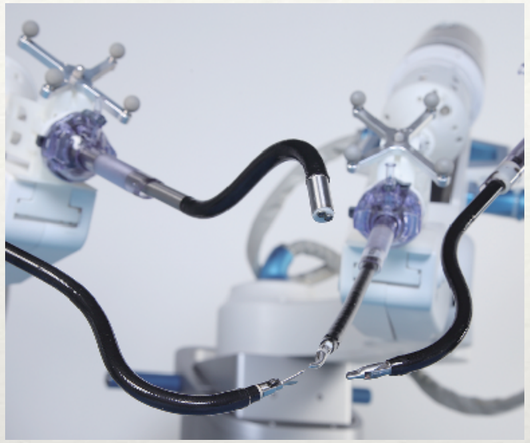
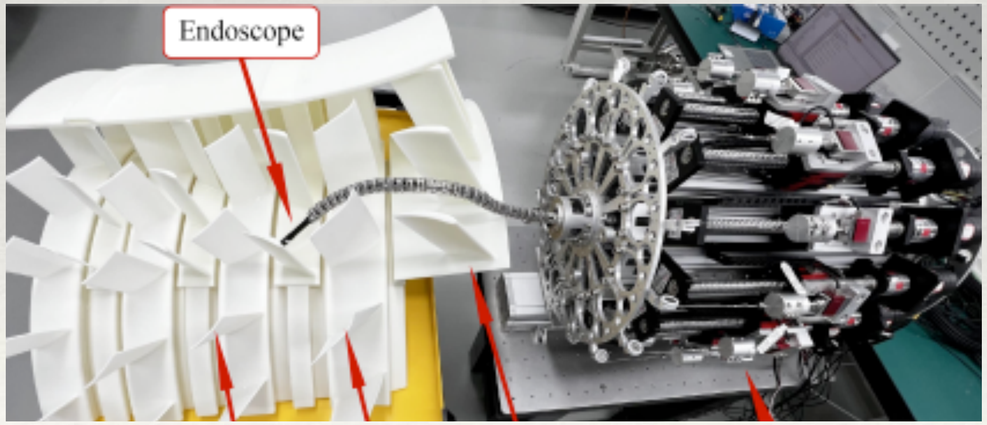
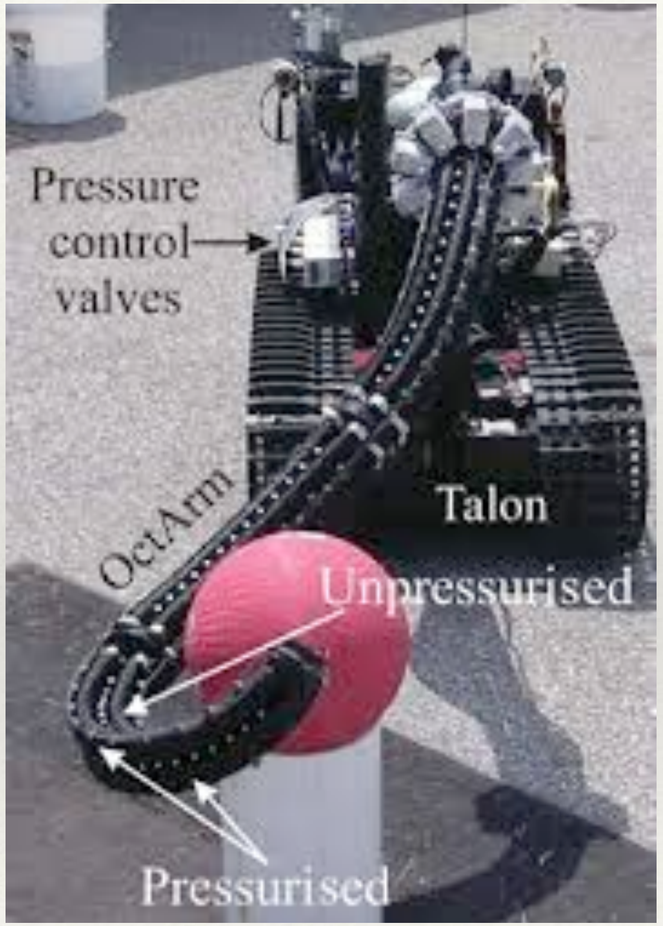
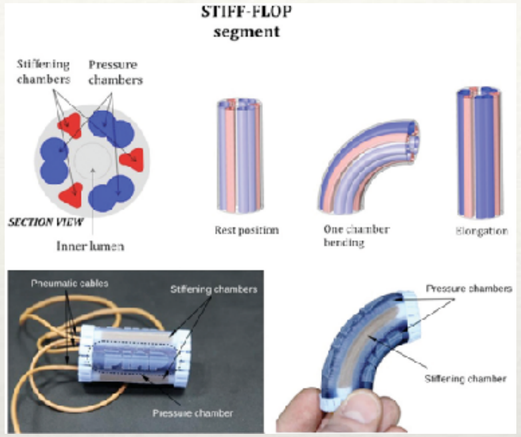
Observation

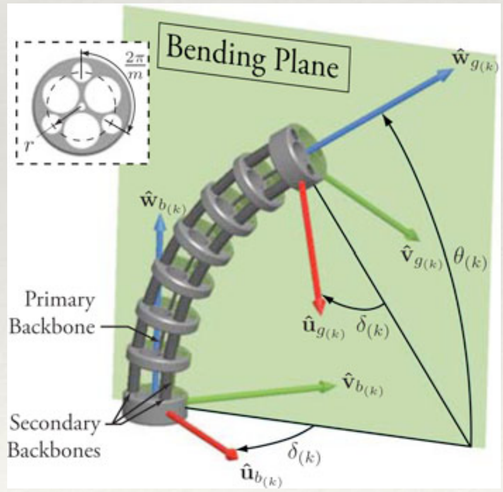


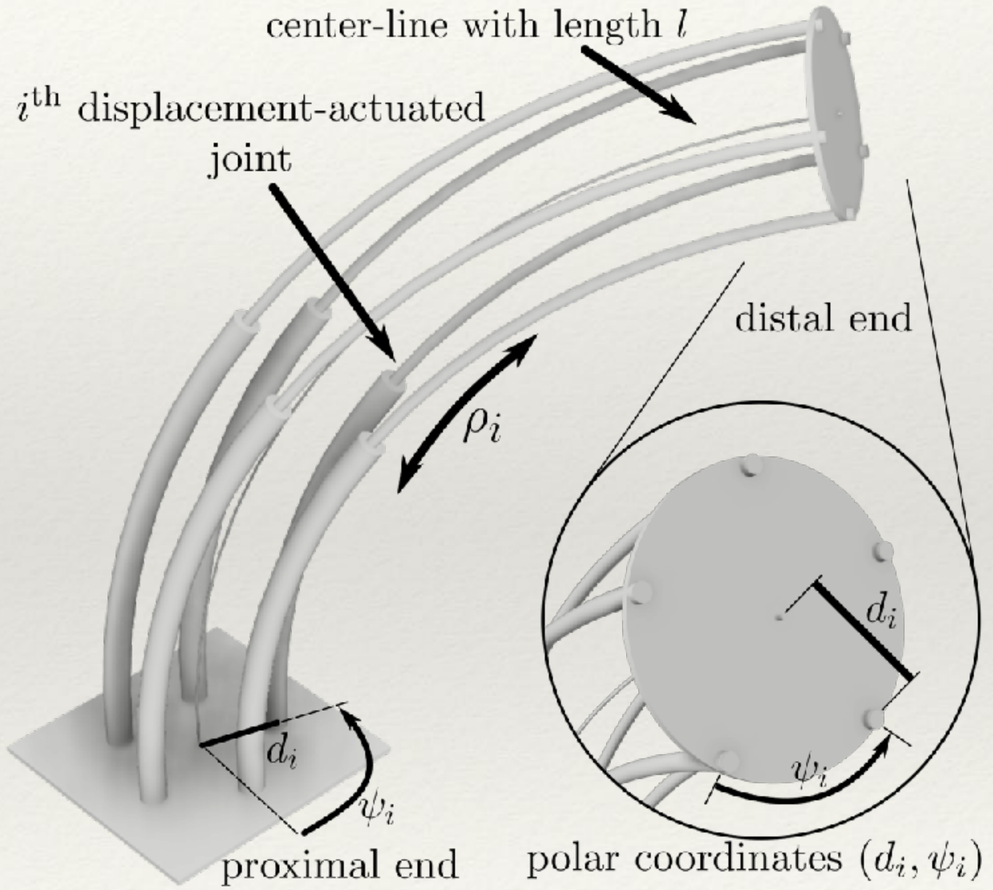




Displacement-Actuated Continuum Robot

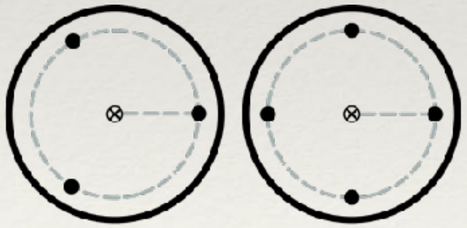






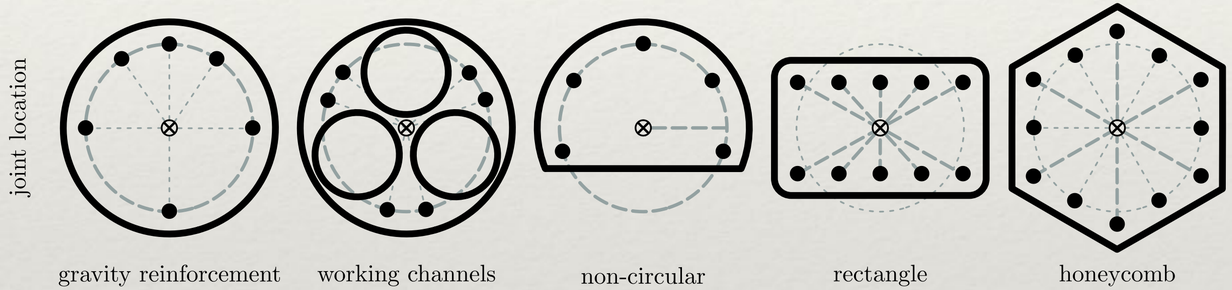
Joint Location

joint location

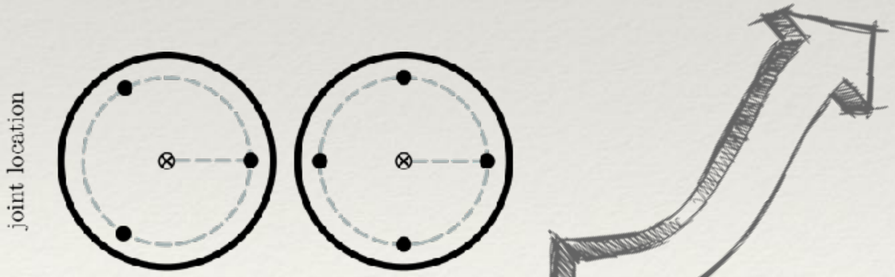


limited option

Joint Location

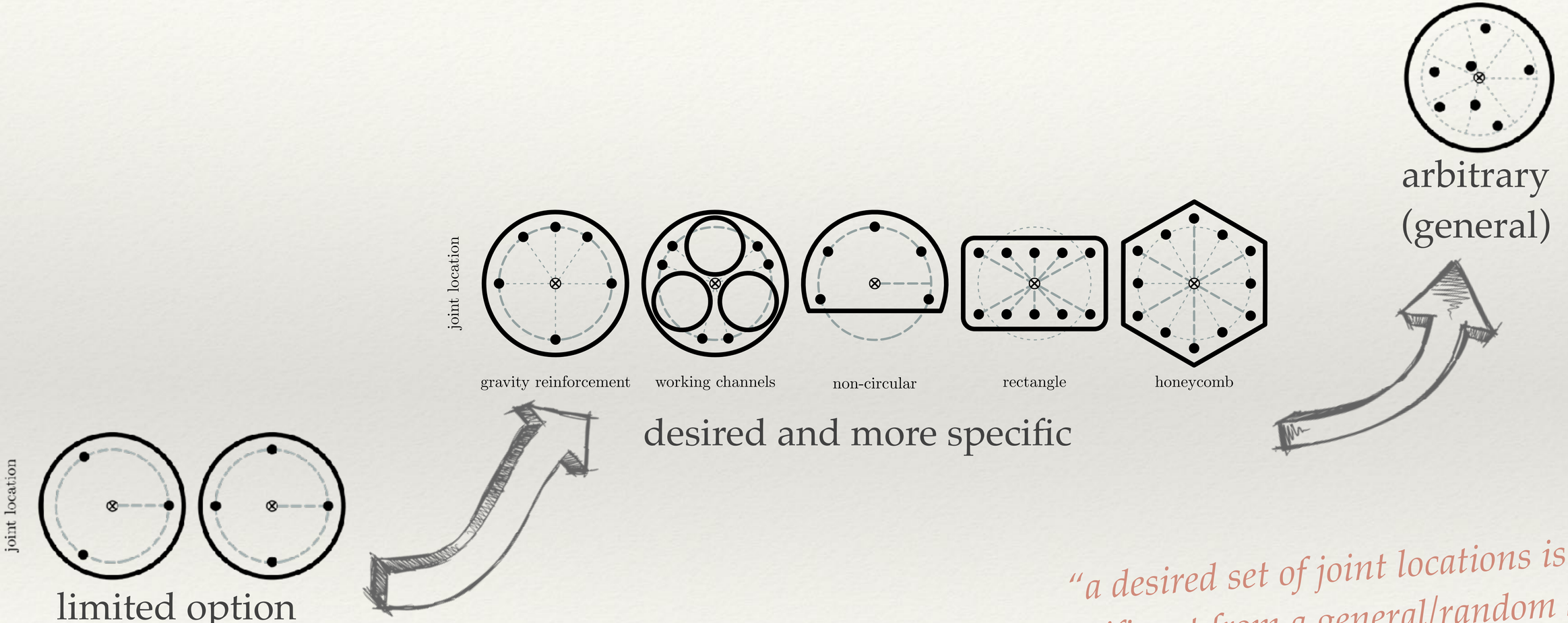


desired and more specific



limited option

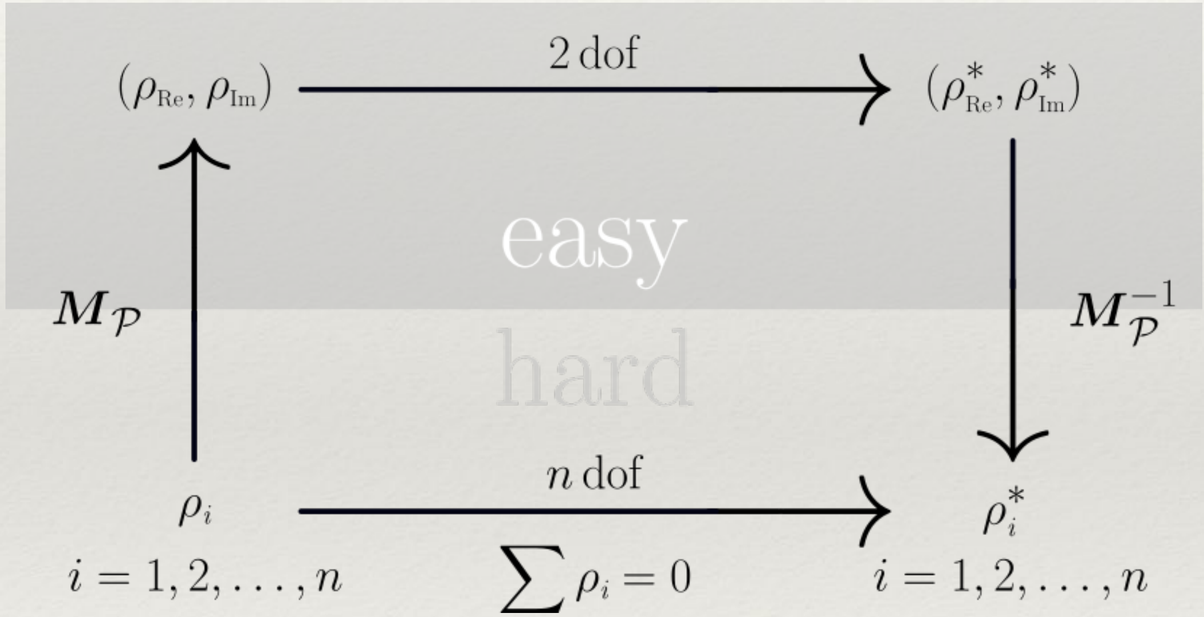
Joint Location



"a desired set of joint locations is a specific set from a general/random set"

Clarke Transform In a Nutshell

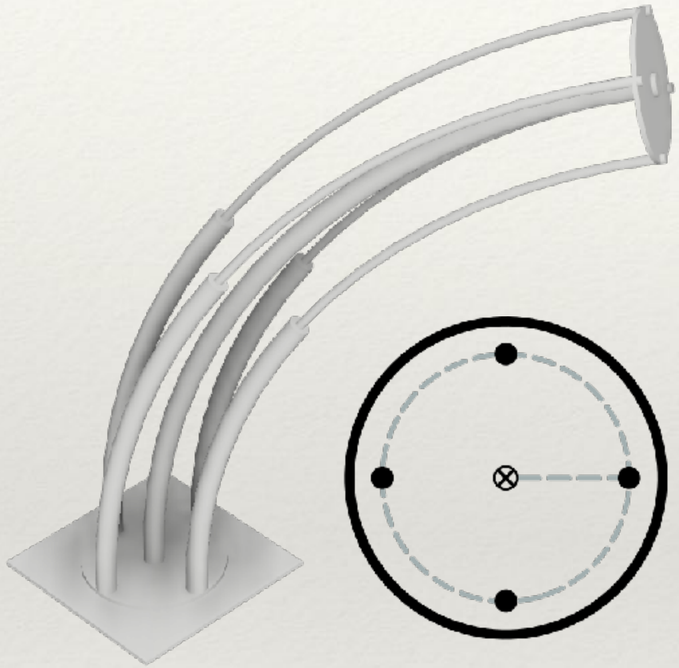
"Similar to Laplace Transform"

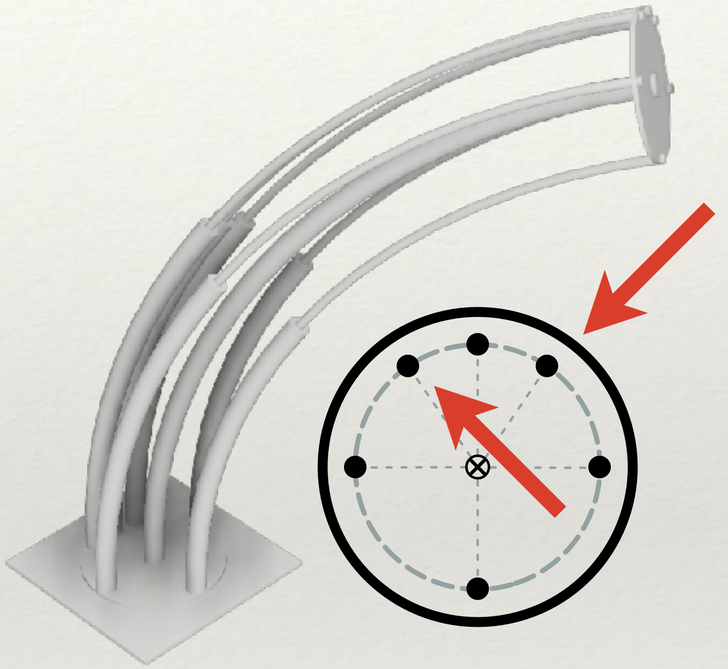


Clarke transform and Clarke coordinates

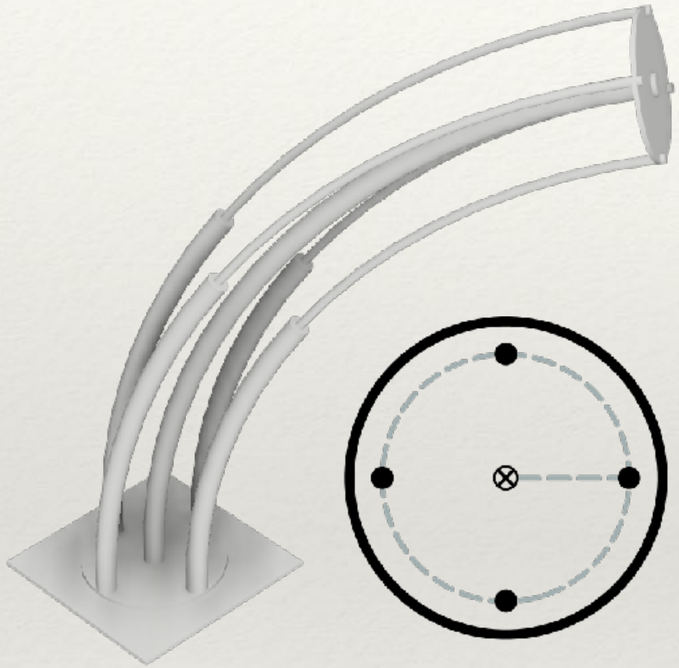
Clarke Transform
(manuscript on arXiv)

Encoder-Decoder Architecture





Encoder-Decoder Architecture



$$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{bmatrix} \subset \mathbb{R}^n$$

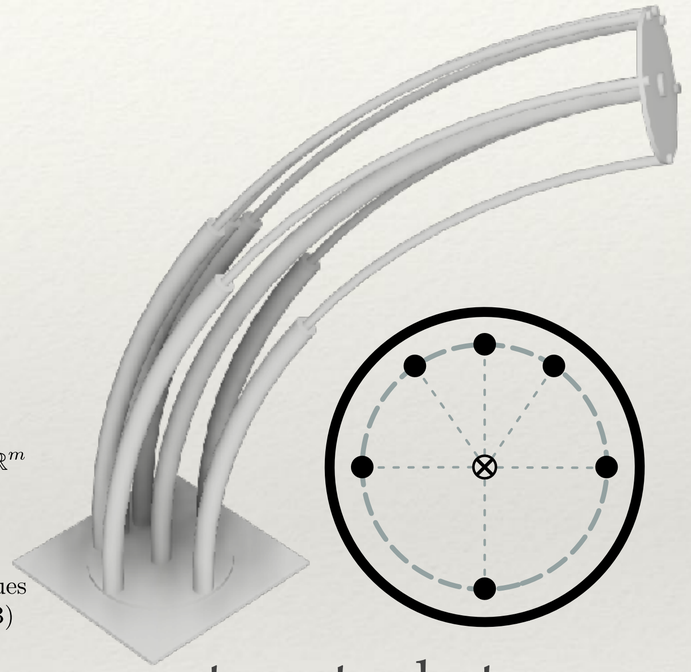
Joint values
(robot A)

surrogate robot
(robot A)

"Boundary condition:
Don't reinvent the wheel"

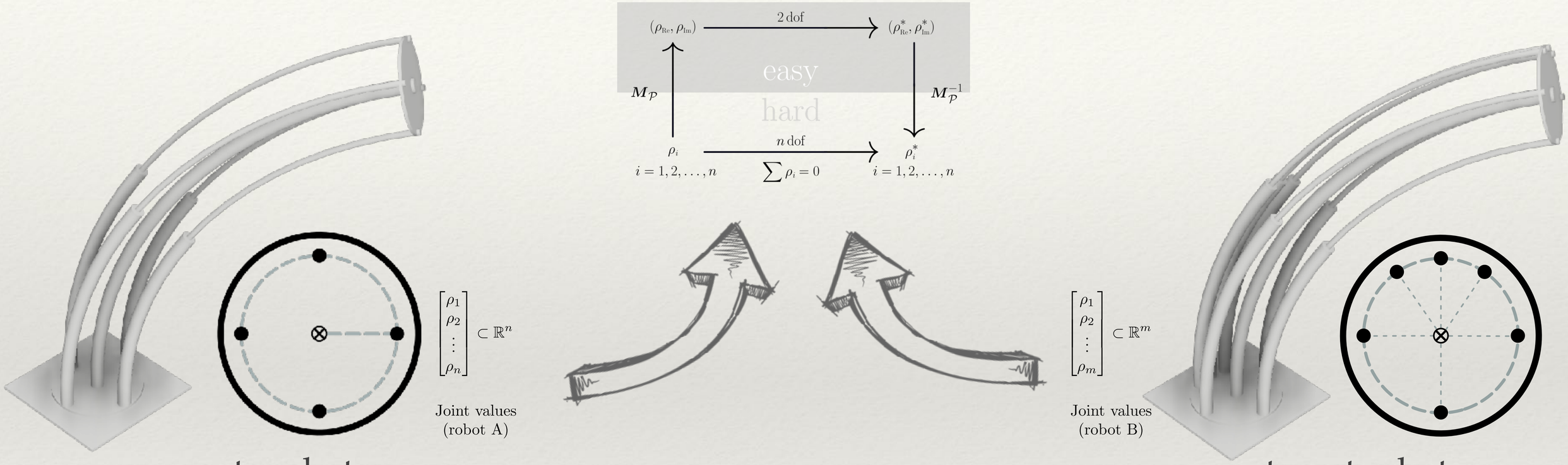
$$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{bmatrix} \subset \mathbb{R}^m$$

Joint values
(robot B)



target robot
(robot B)

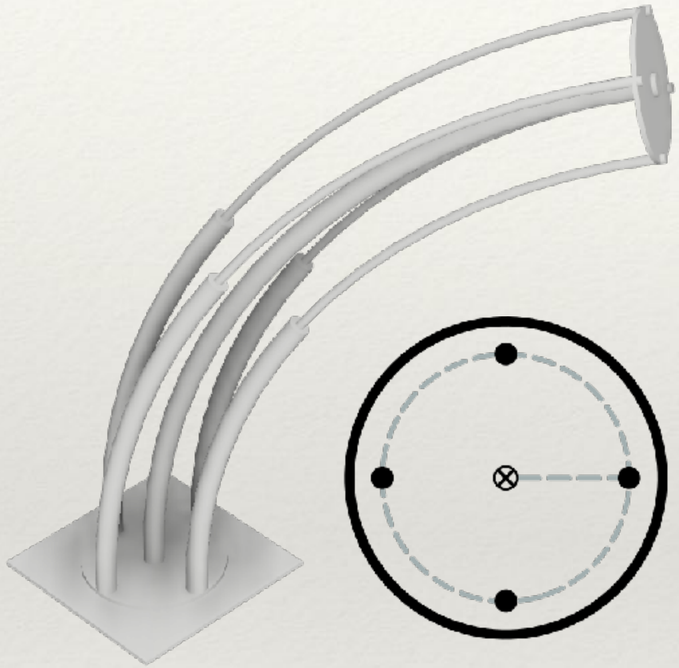
Encoder-Decoder Architecture



surrogate robot (robot A)

target robot (robot B)

Encoder-Decoder Architecture



$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{bmatrix} \subset \mathbb{R}^n$

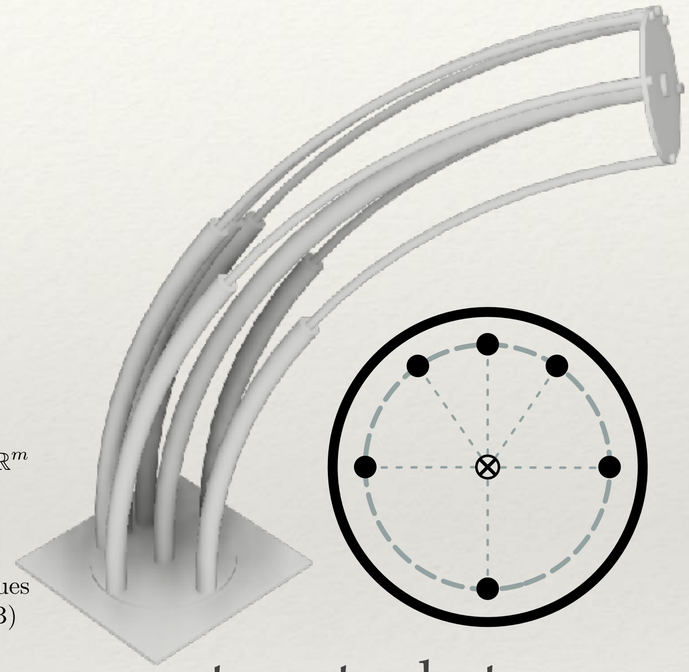
Joint values
(robot A)

$\begin{bmatrix} \rho_{\text{Re}} \\ \rho_{\text{Im}} \end{bmatrix} \in \mathbb{R}^2$

Clarke coordinates

$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{bmatrix} \subset \mathbb{R}^m$

Joint values
(robot B)



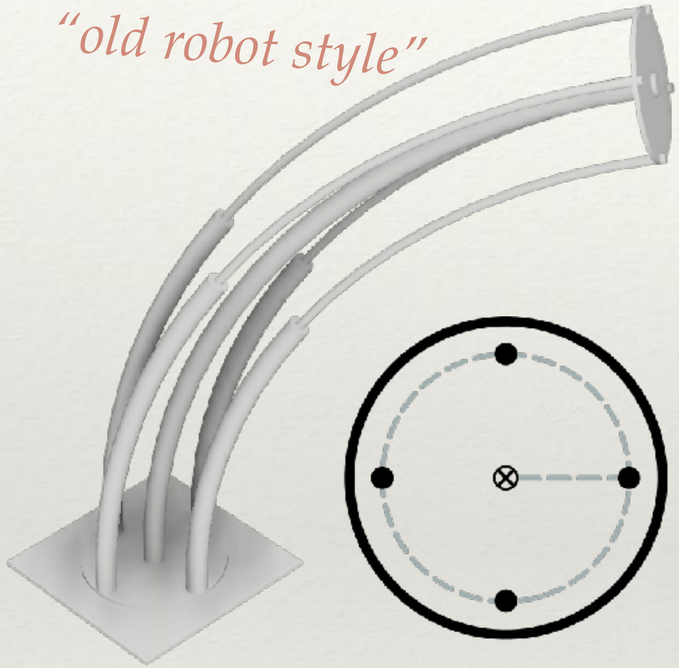
surrogate robot
(robot A)

shared manifold

target robot
(robot B)

Encoder-Decoder Architecture

"old robot style"



surrogate robot
(robot A)

$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{bmatrix} \subset \mathbb{R}^n$

Joint values
(robot A)

Encoder

Forward Clarke Transform
(robot A)

$\begin{bmatrix} \rho_{\text{Re}} \\ \rho_{\text{Im}} \end{bmatrix} \in \mathbb{R}^2$

Clarke coordinates

Decoder

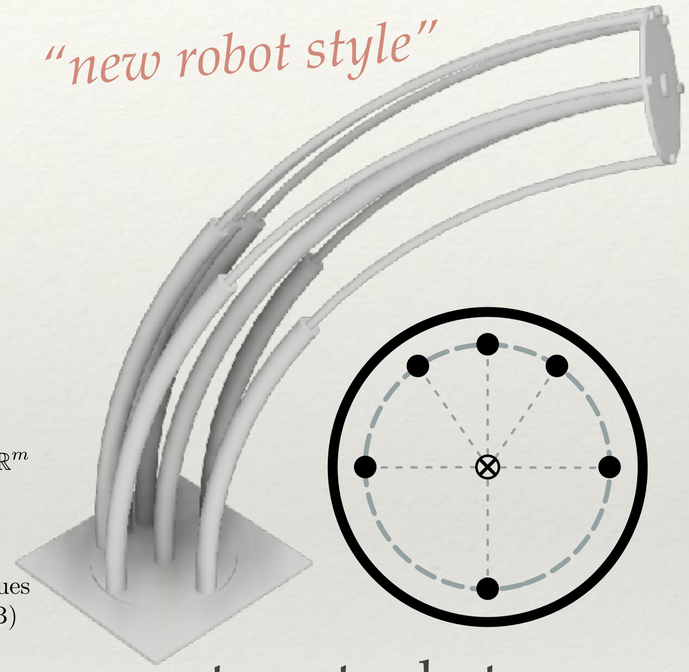
Inverse Clarke Transform
(robot B)

$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{bmatrix} \subset \mathbb{R}^m$

Joint values
(robot B)

shared manifold

"new robot style"

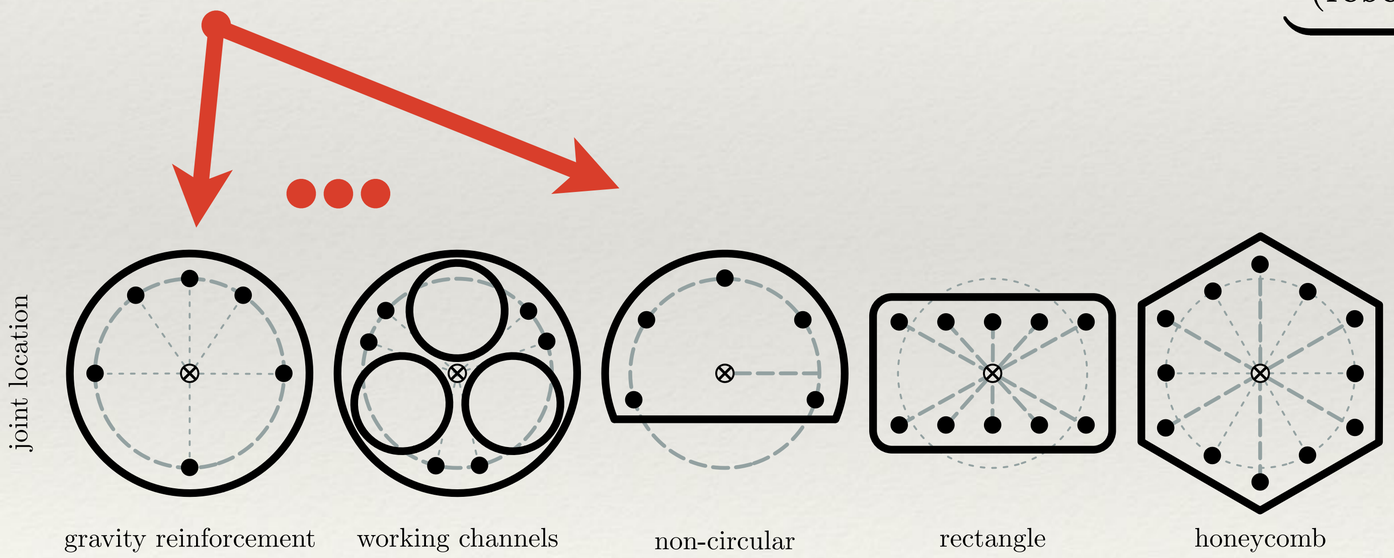


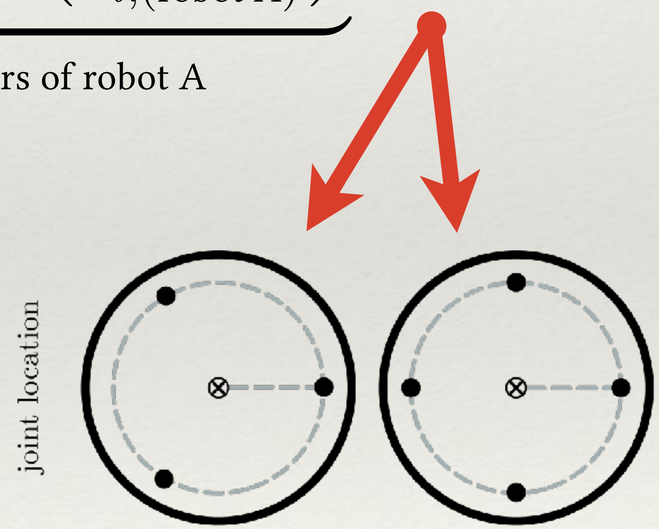
target robot
(robot B)

"think of style transfer"

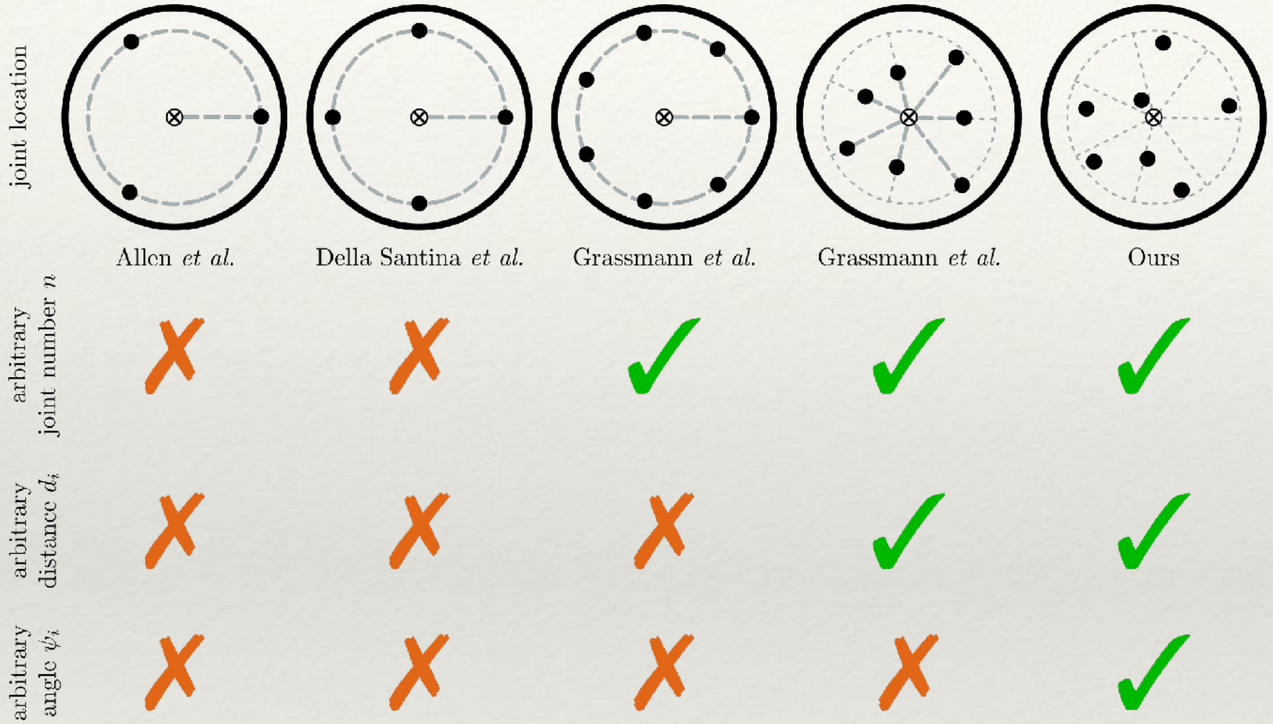
Closed-Form, Compact, and Interpretable

$$\boldsymbol{\rho}_{(\text{robot B})} = \overbrace{l_{(\text{robot B})}\,\mathrm{diag}\left(d_{i,(\text{robot B})}\right)\boldsymbol{M}_{\mathcal{P}}^{-1}{}_{(\text{robot B})}}^{\text{adds design parameters of robot B}} \cdot \underbrace{\frac{1}{l_{(\text{robot A})}}\boldsymbol{M}_{\mathcal{P}\,(\text{robot A})}\,\mathrm{diag}\left(\frac{1}{d_{i,(\text{robot A})}}\right)}_{\text{removes design parameters of robot A}}\boldsymbol{\rho}_{(\text{robot A})}$$

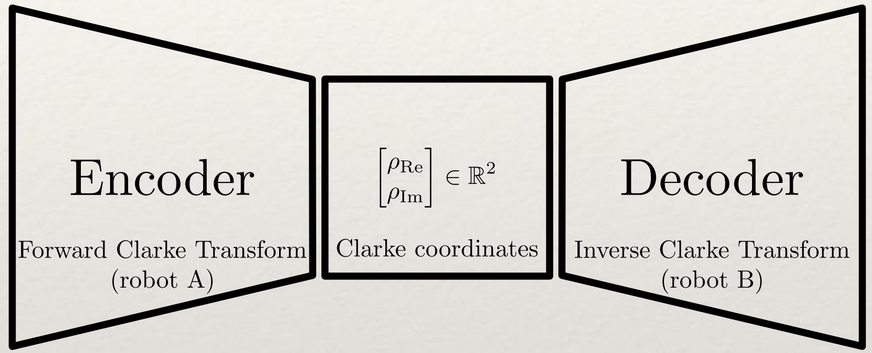




Contribution



arbitrary joint locations for desired joint locations



method for re-using previous methods

NSERC CRSNG

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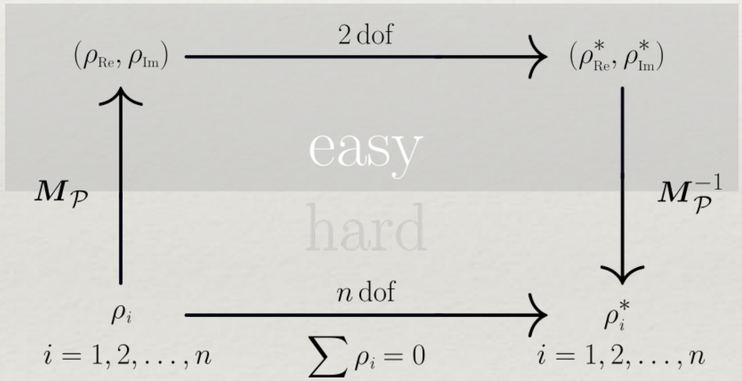
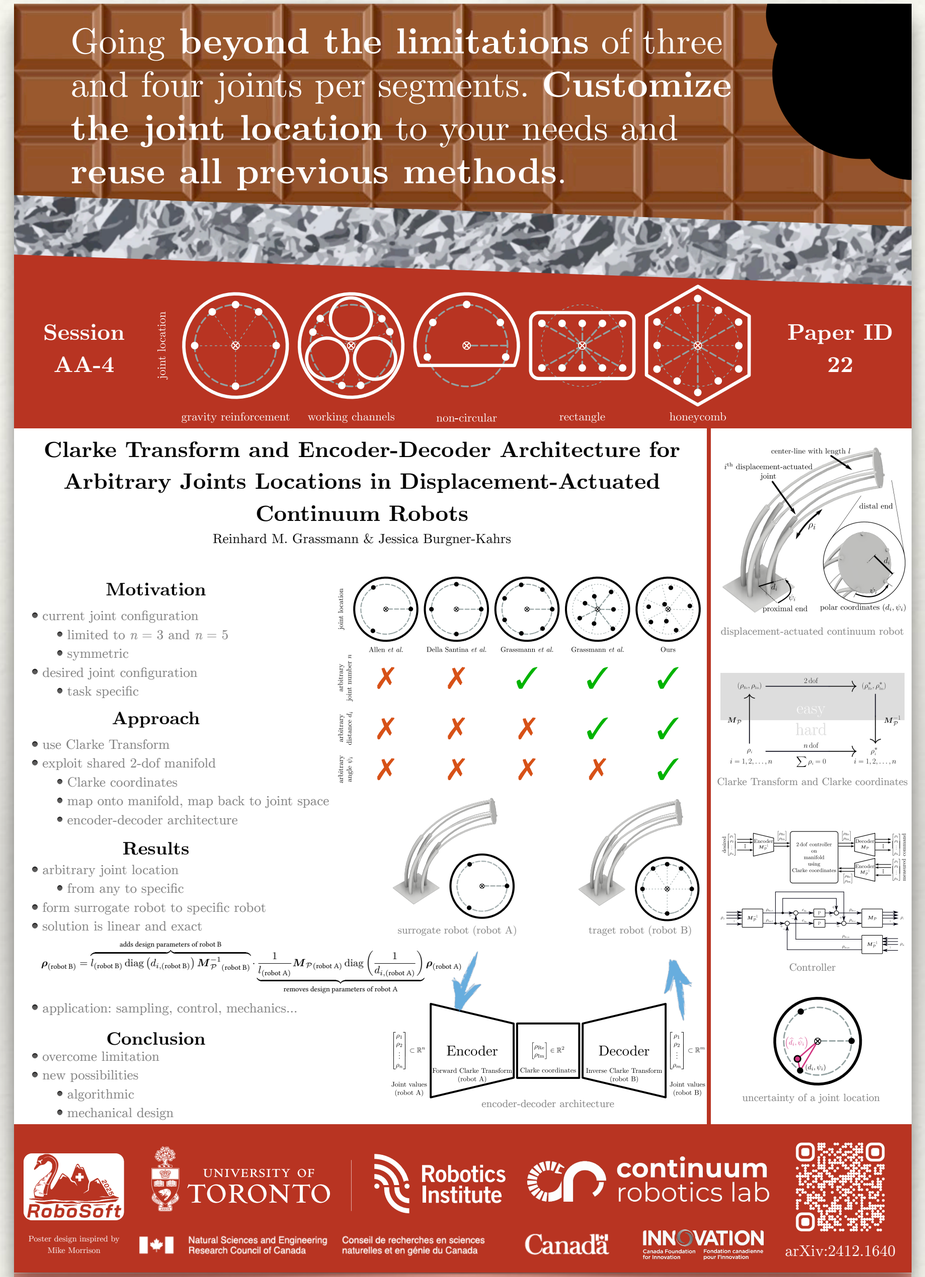
Thank you for your time and attention



Paper ID 22

Call for Action:
Use Clarke Transform



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