

RobotSoft 2025

Clarke Transform and Encoder-Decoder Architecture for Arbitrary Joint Locations in Displacement-Actuated Continuum Robots

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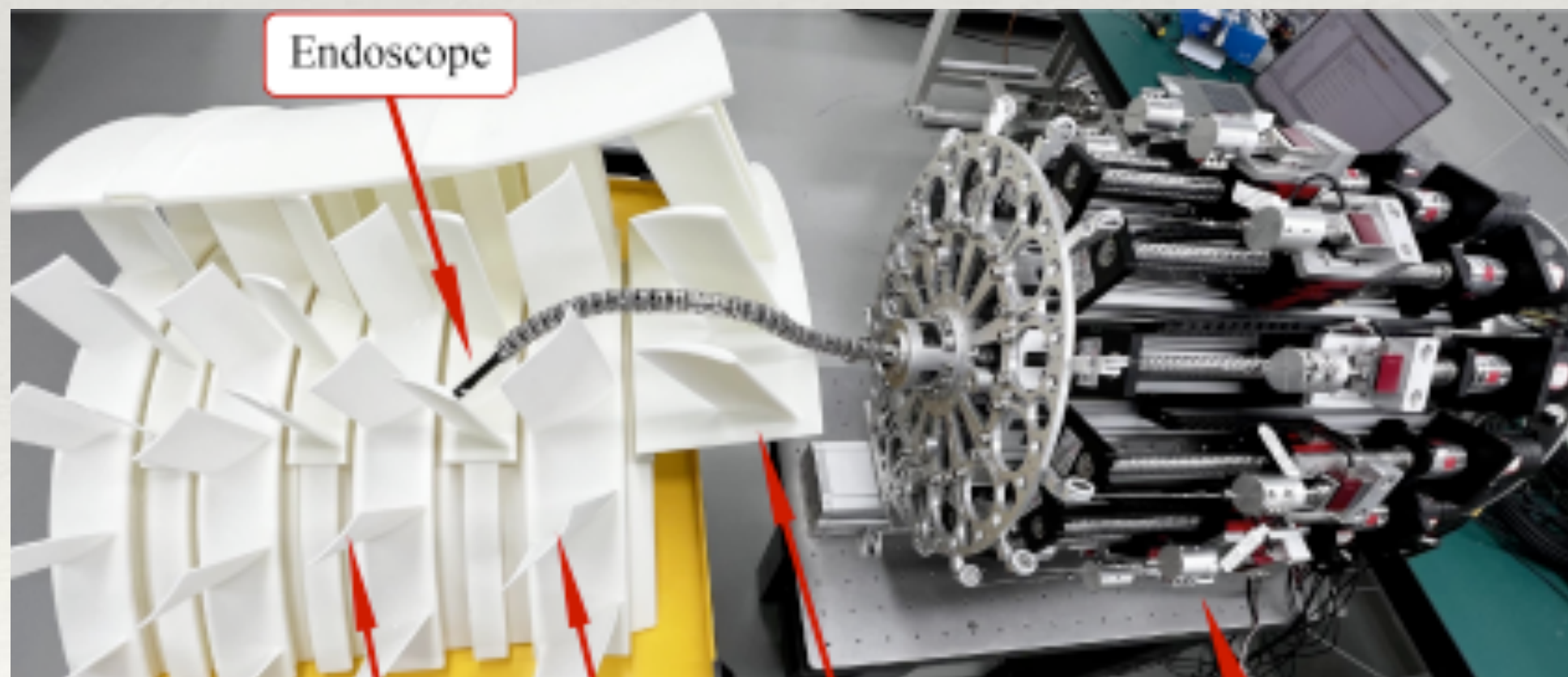
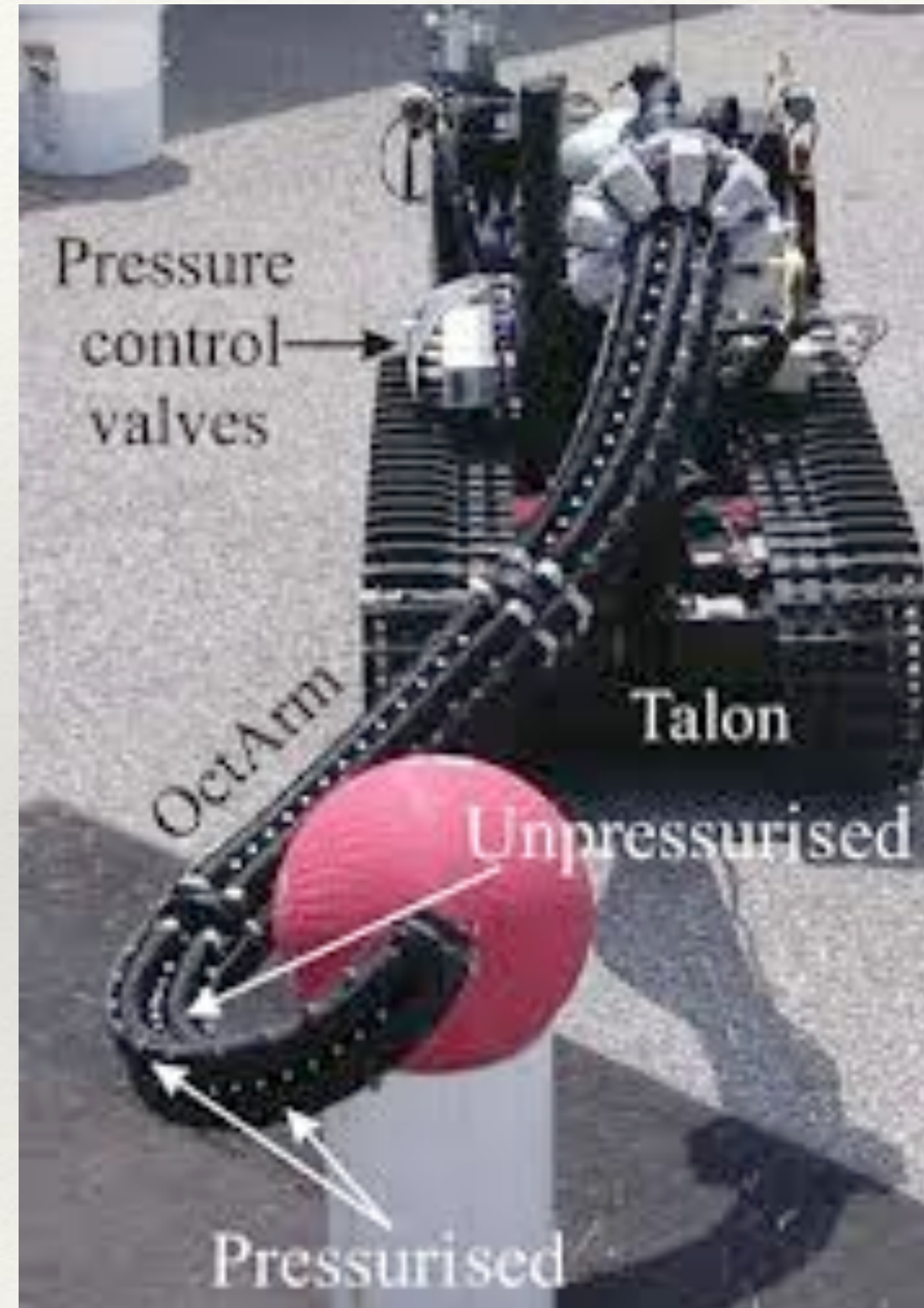
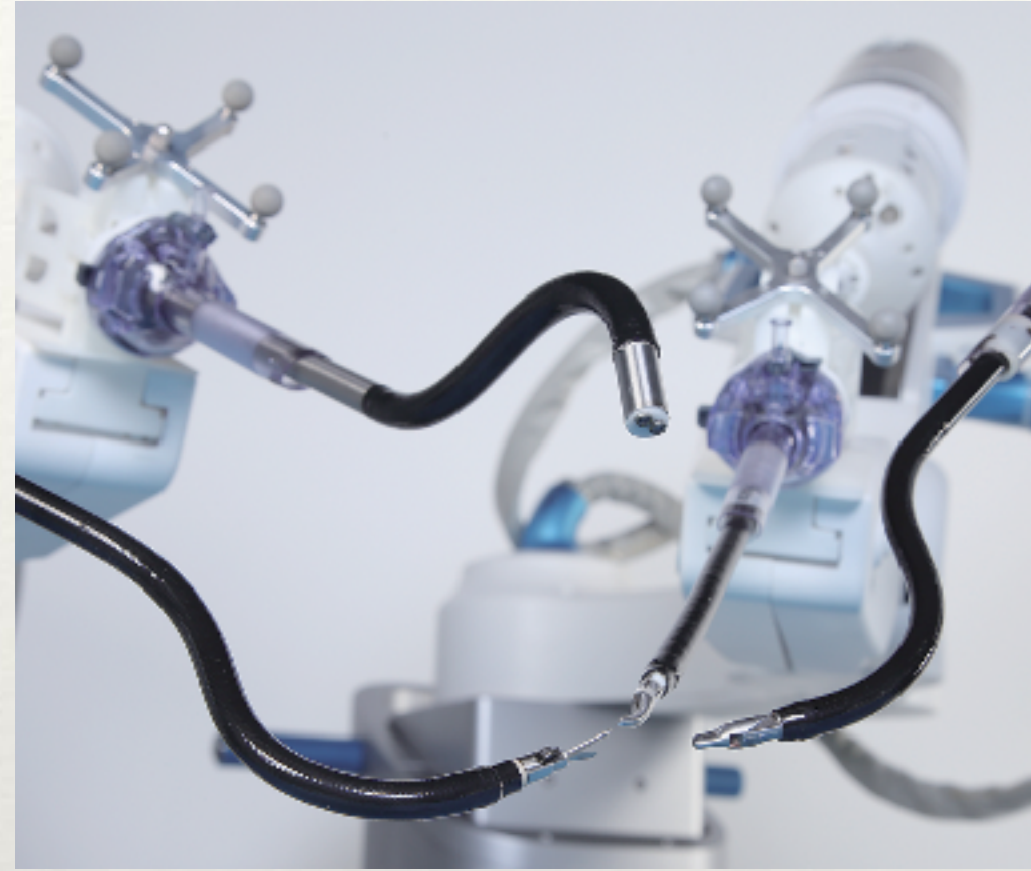
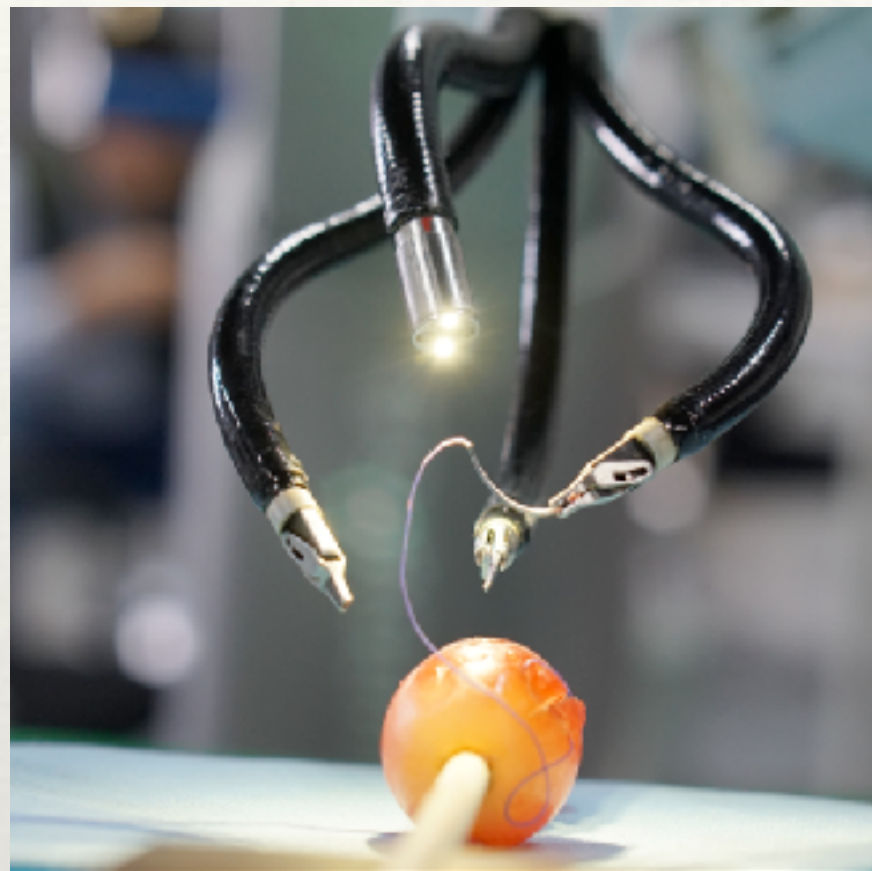


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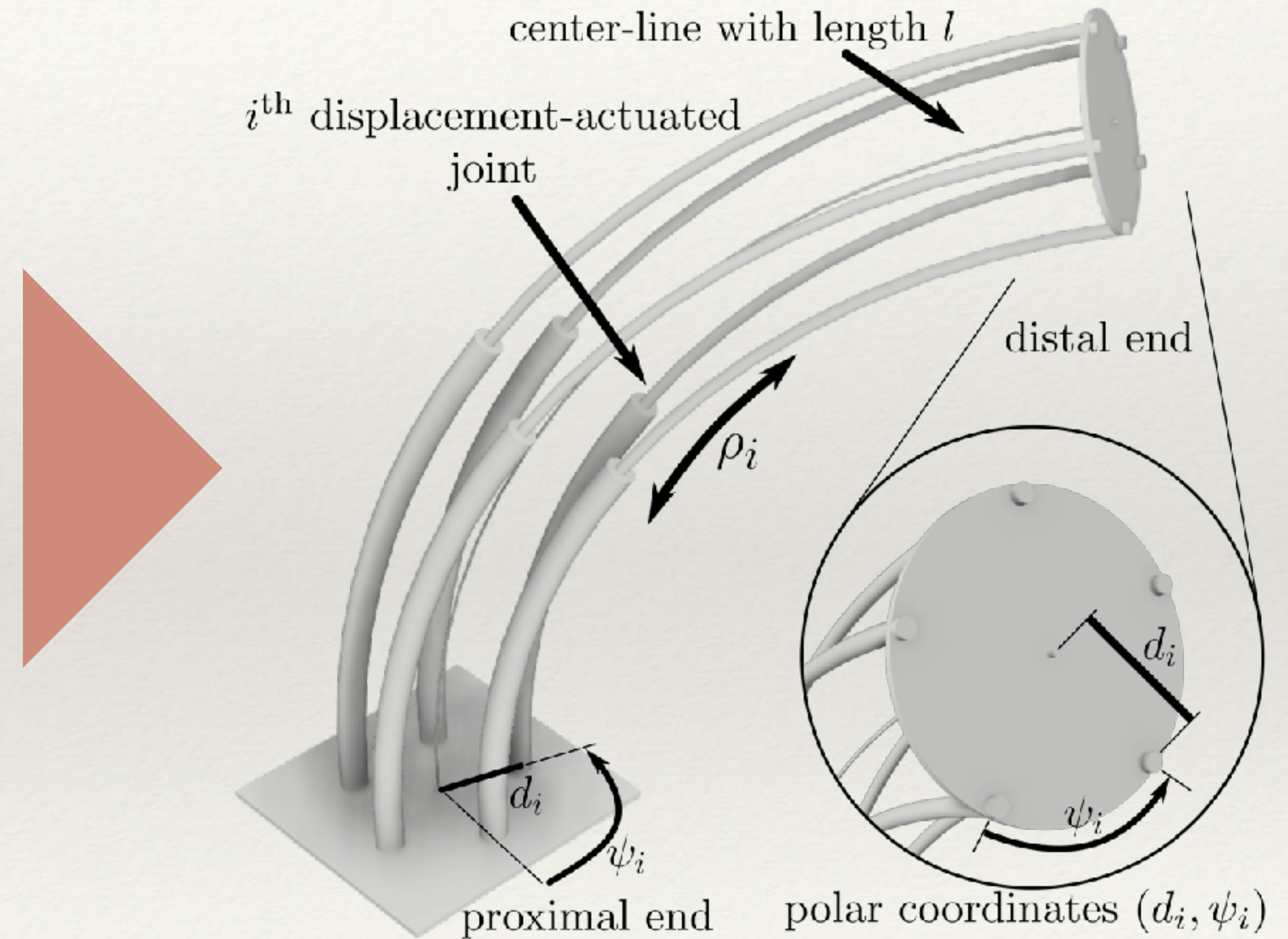
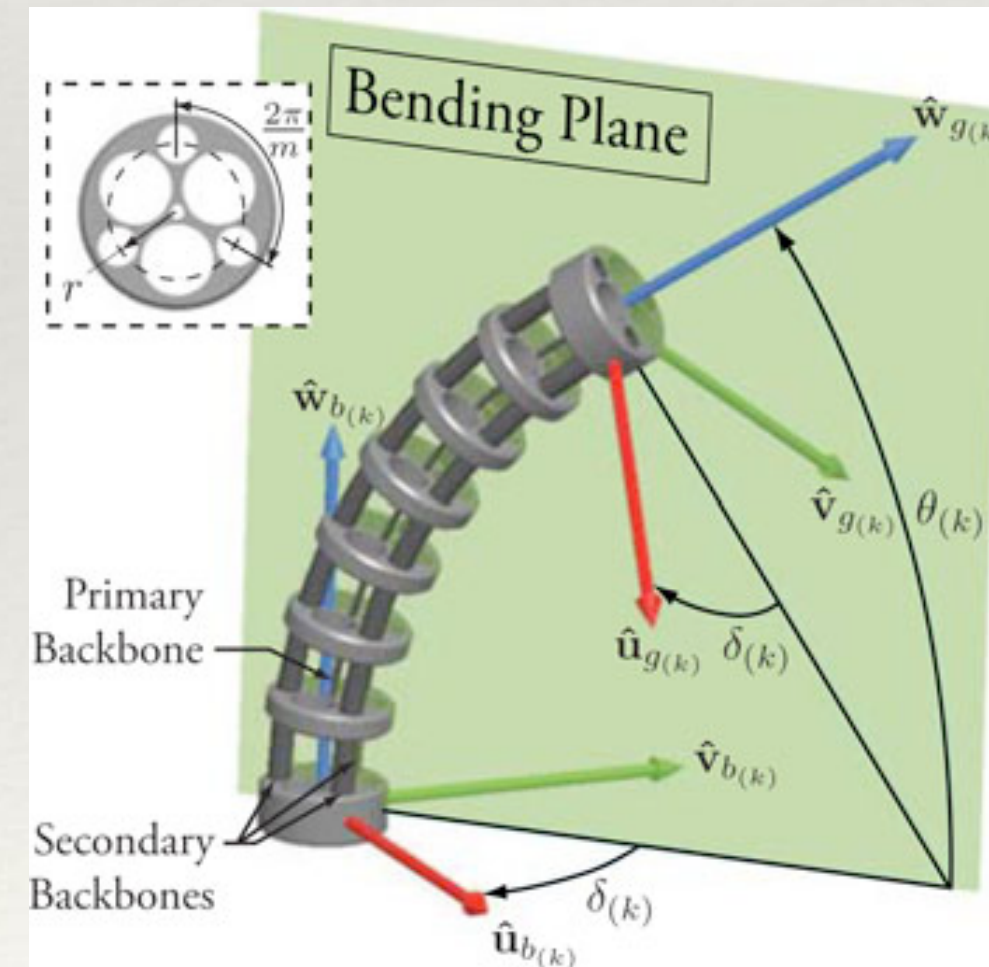
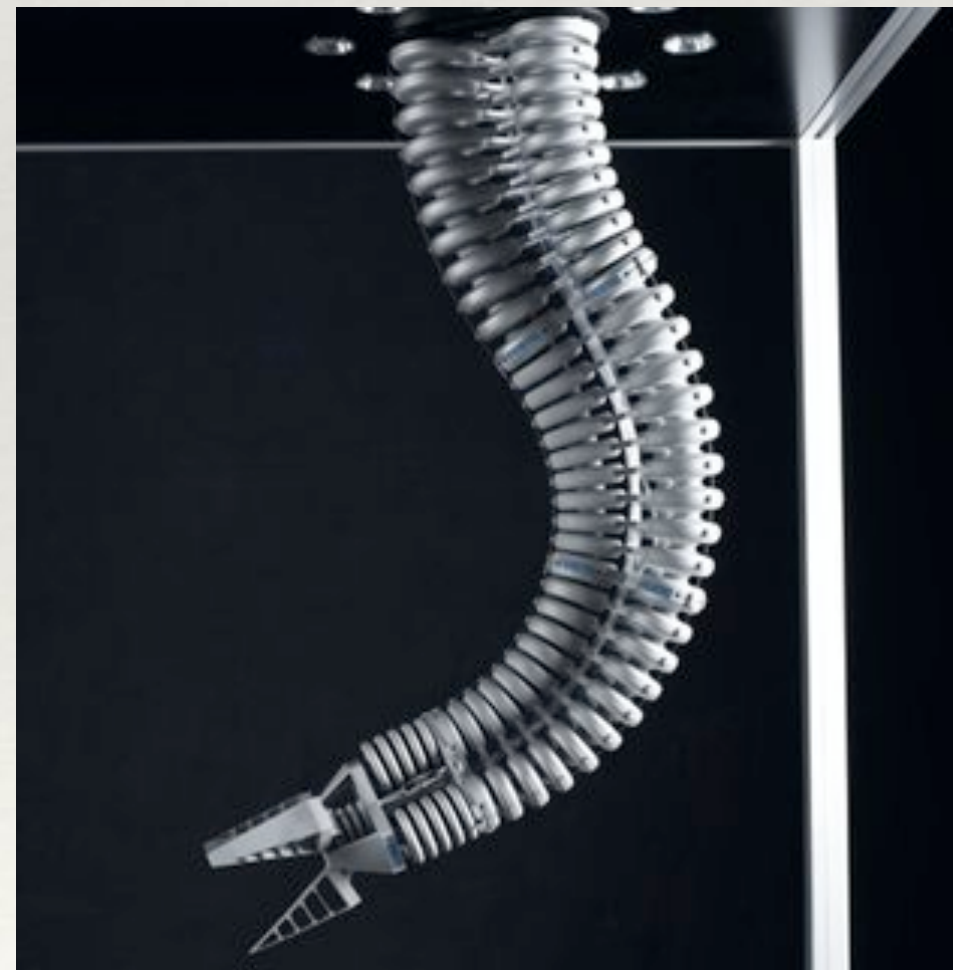
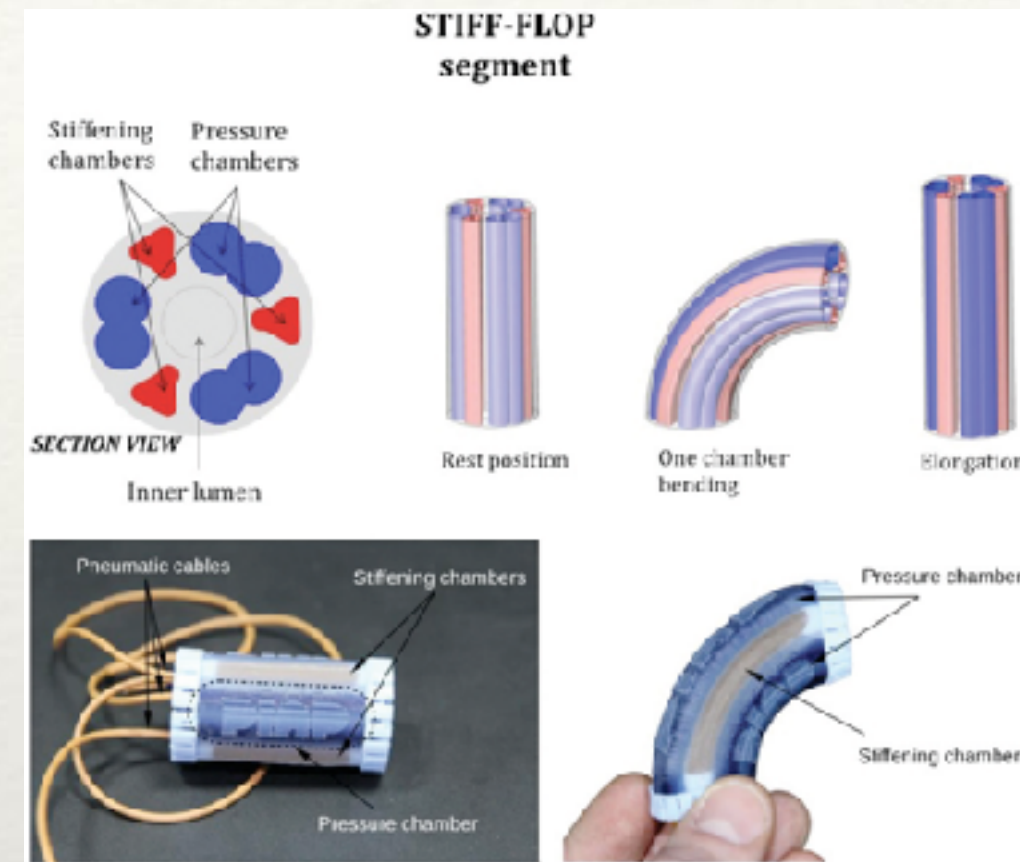


continuum
robotics lab

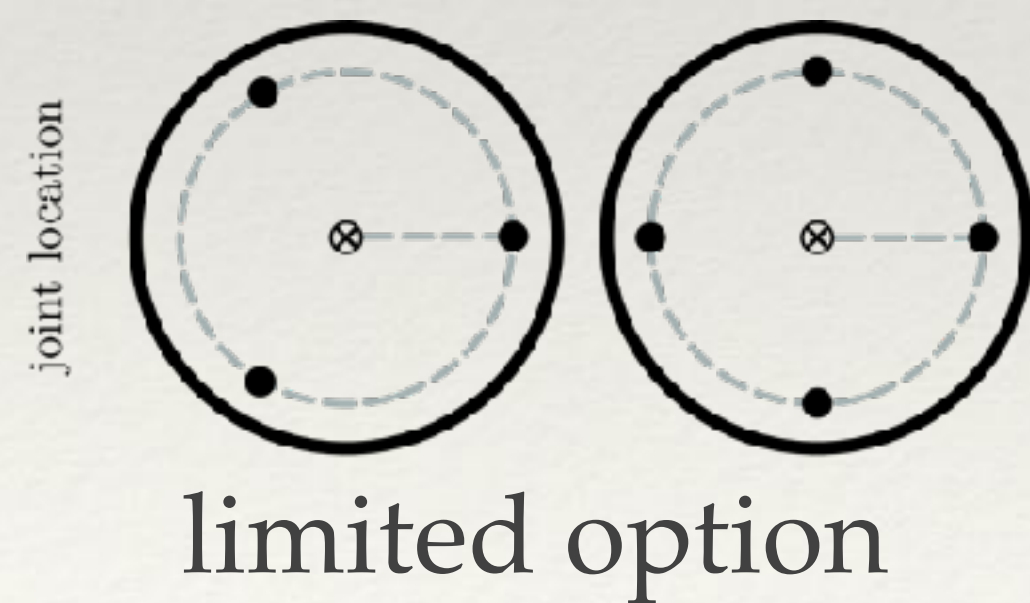
Observation



Displacement-Actuated Continuum Robot



Joint Location



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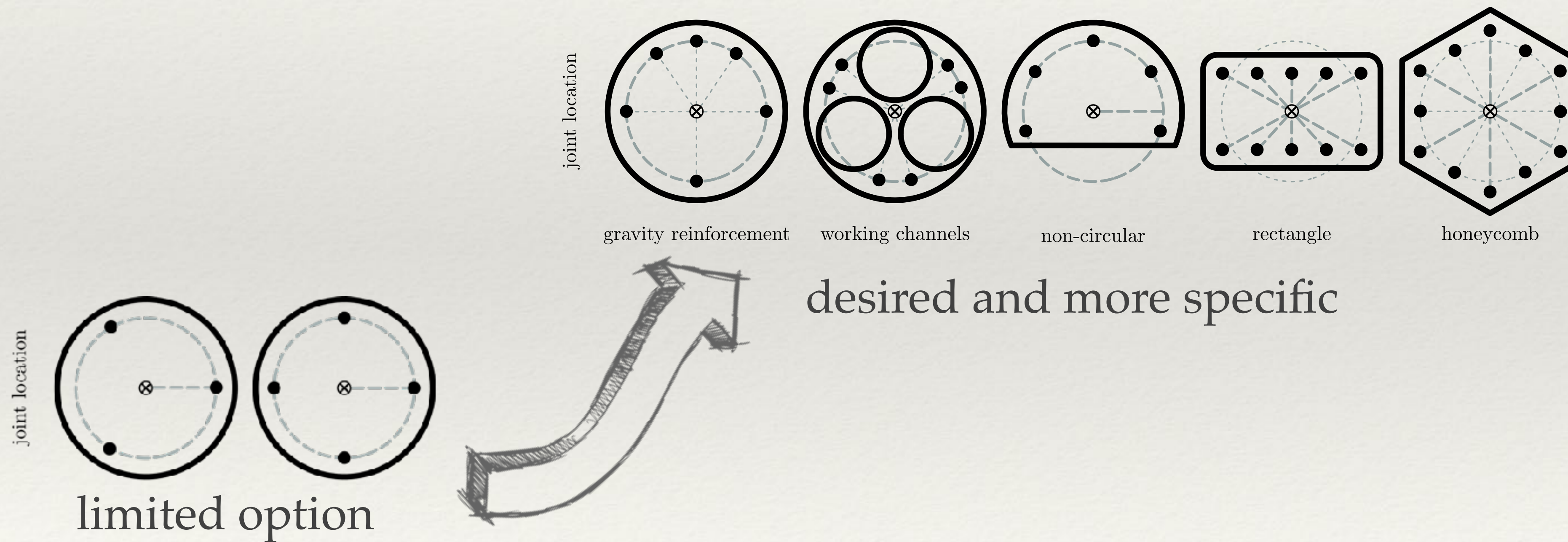


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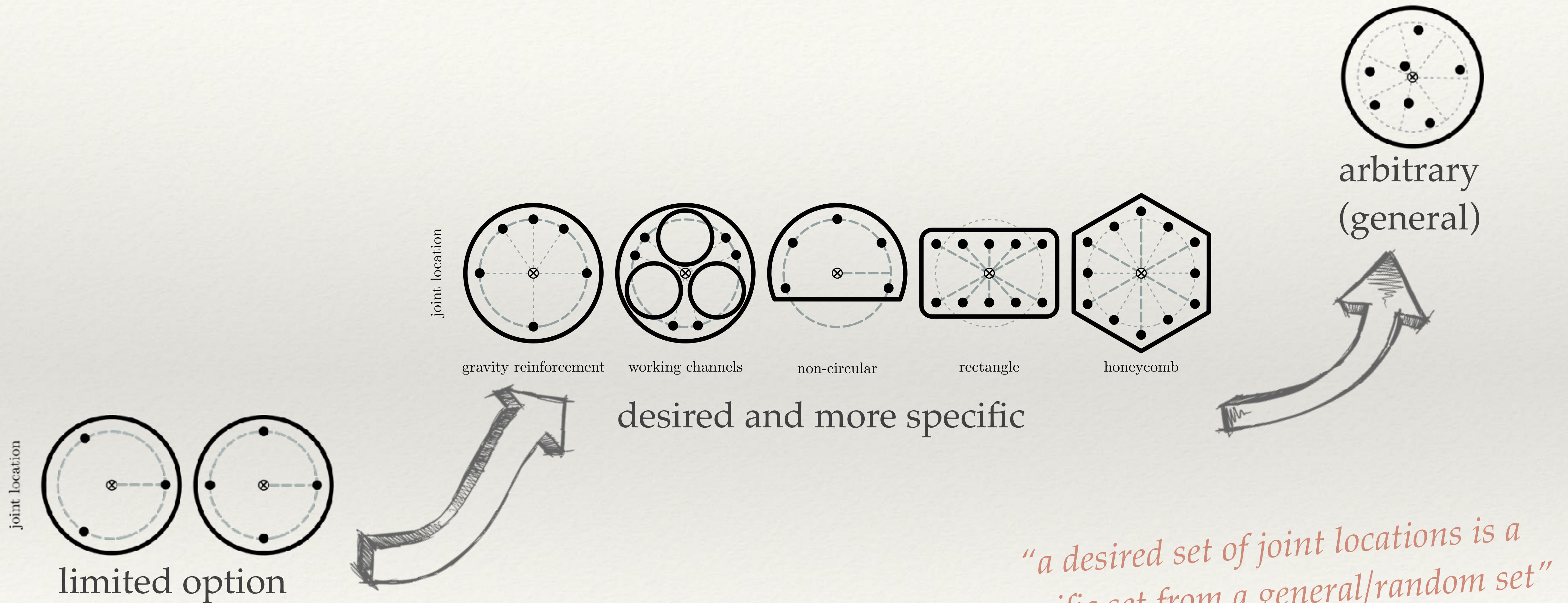


continuum
robotics lab

Joint Location



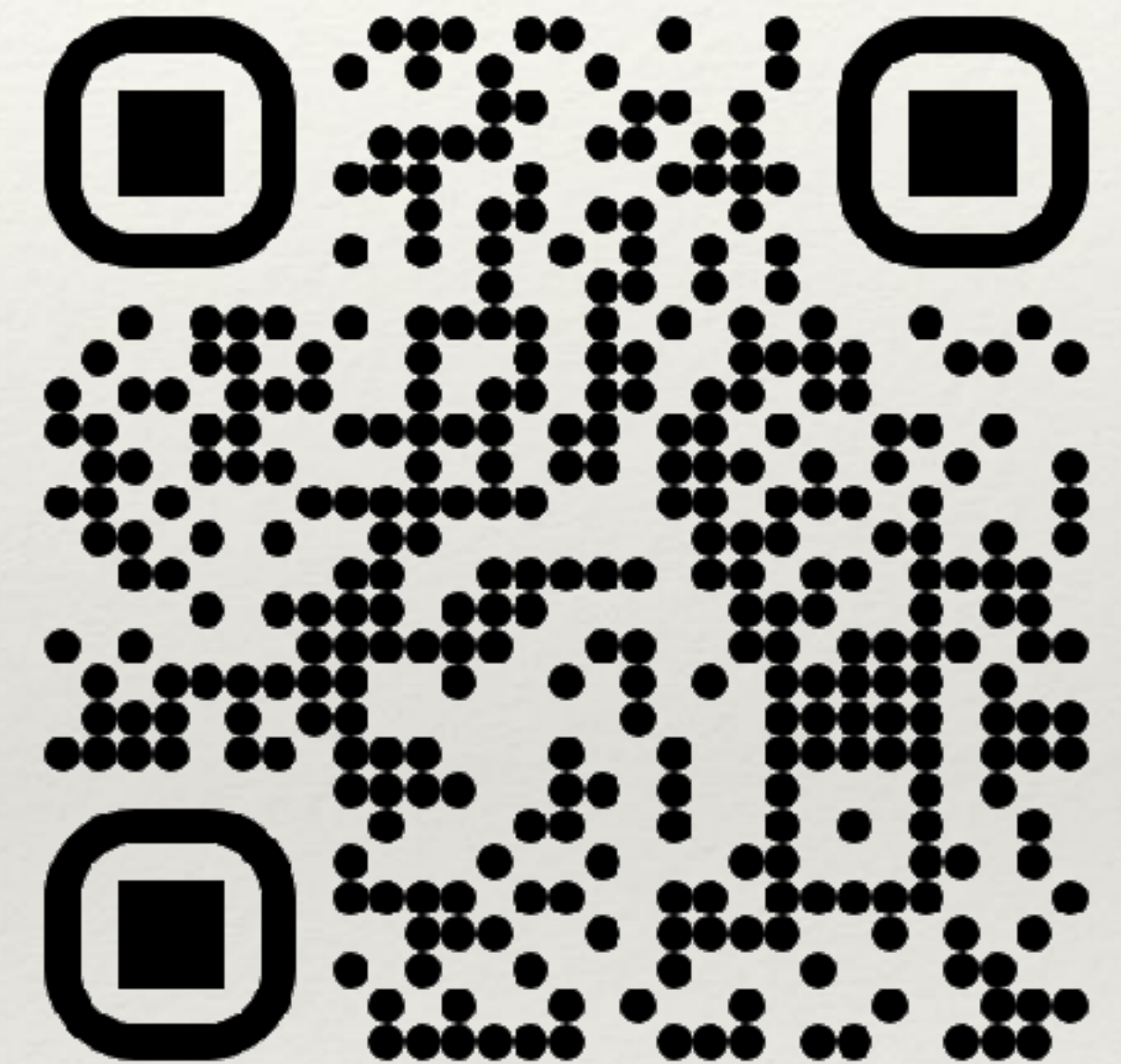
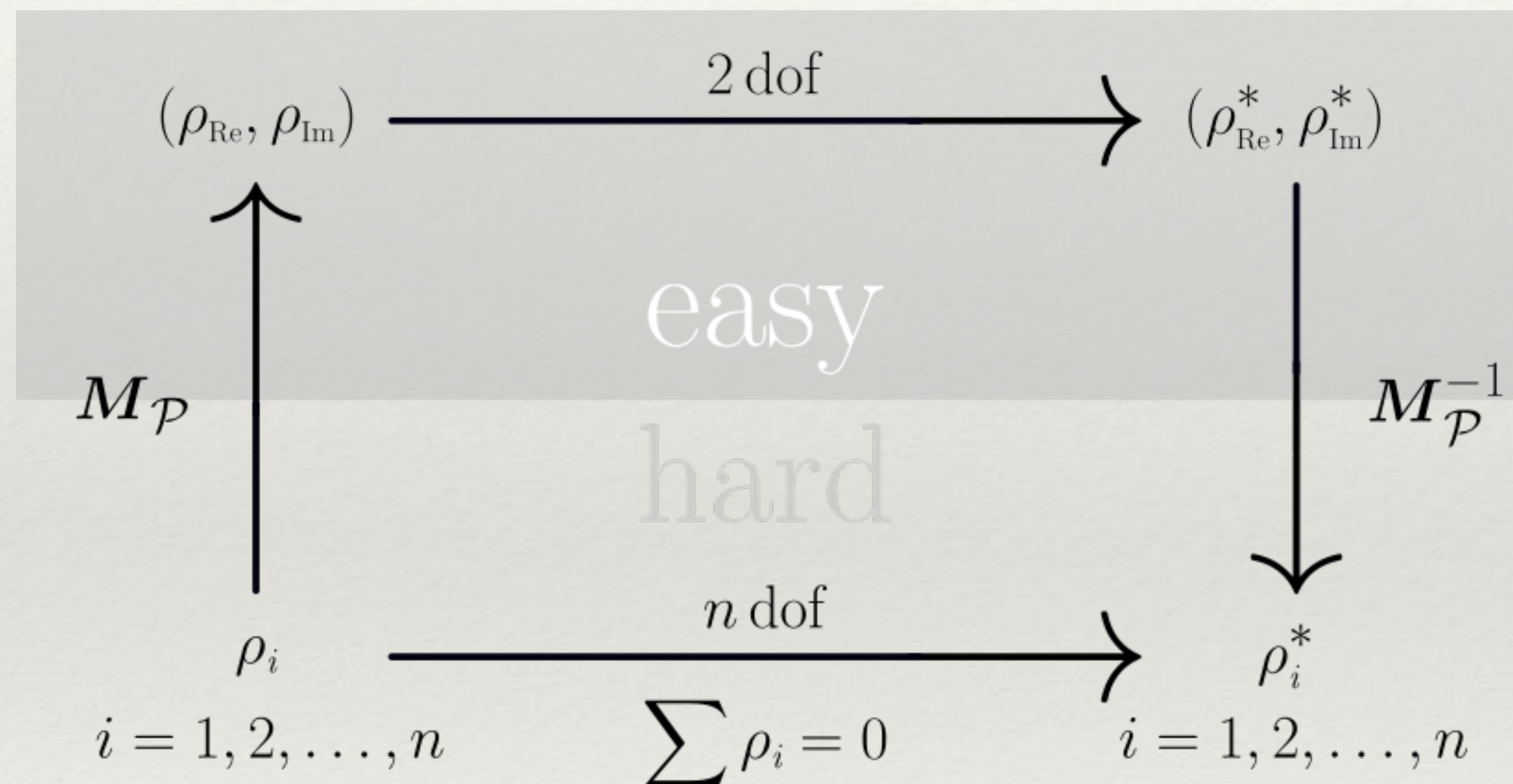
Joint Location



"a desired set of joint locations is a specific set from a general/random set"

Clarke Transform In a Nutshell

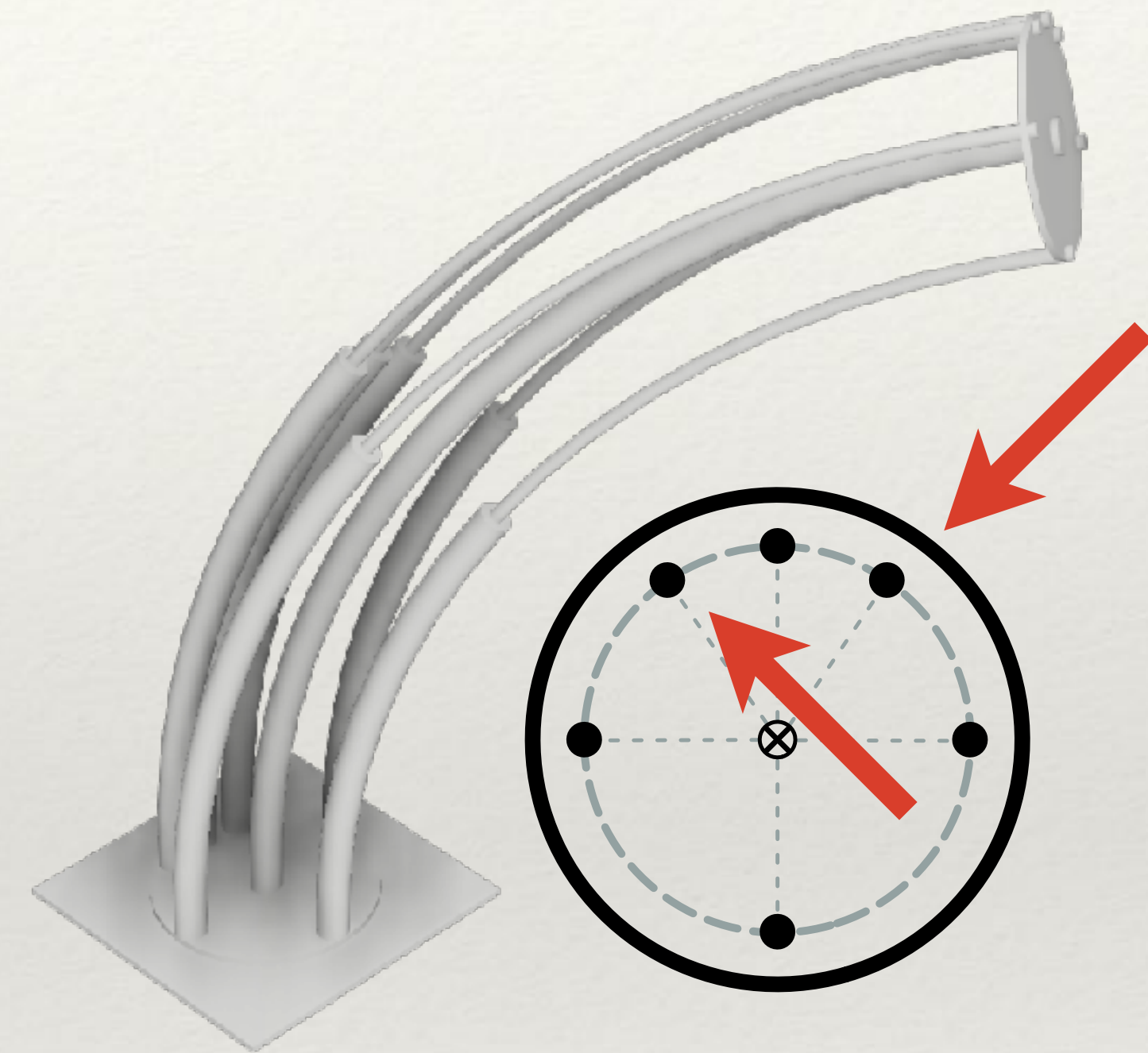
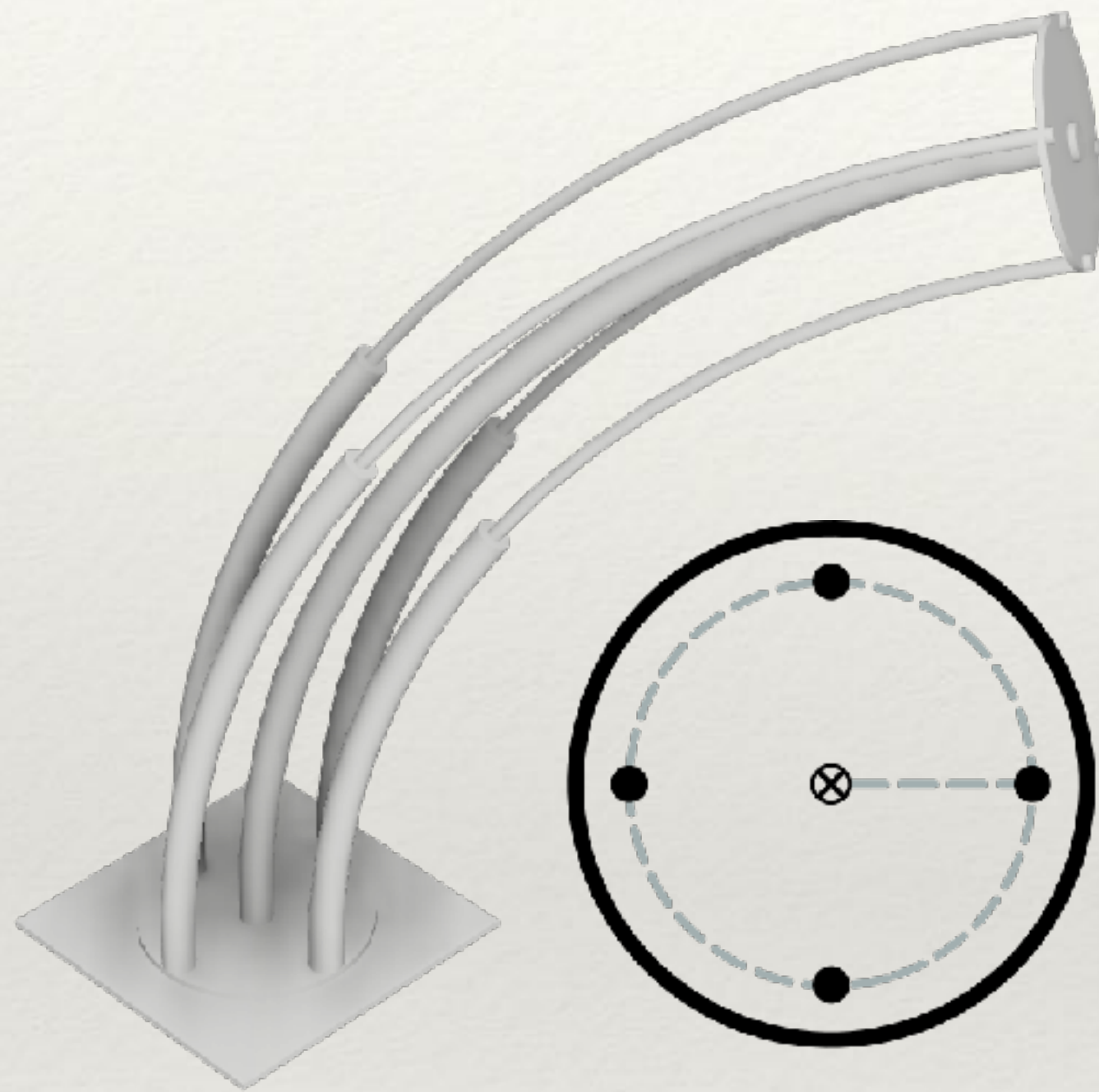
“Similar to Laplace Transform”



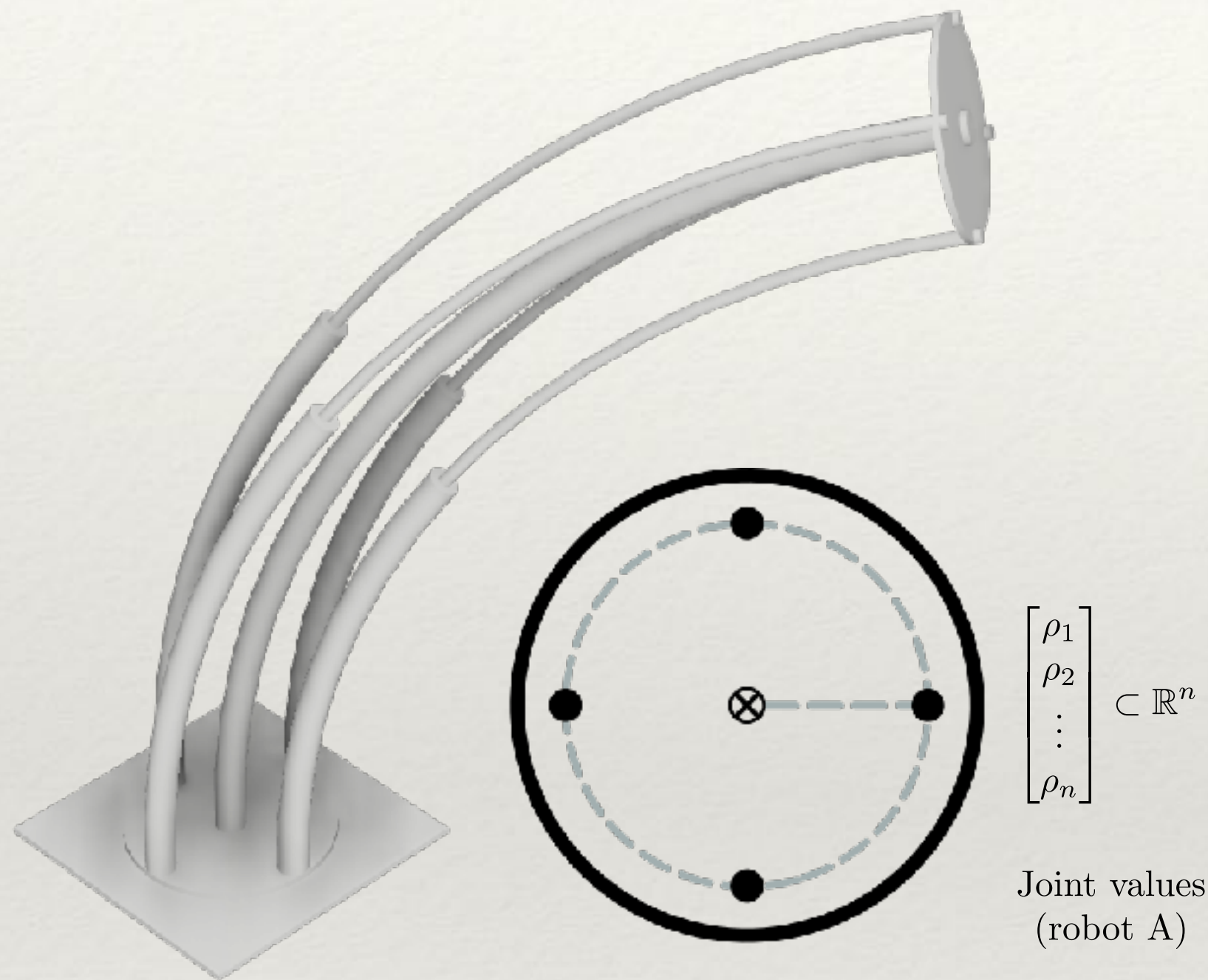
Clarke Transform
(manuscript on arXiv)

Clarke transform and Clarke coordinates

Encoder-Decoder Architecture

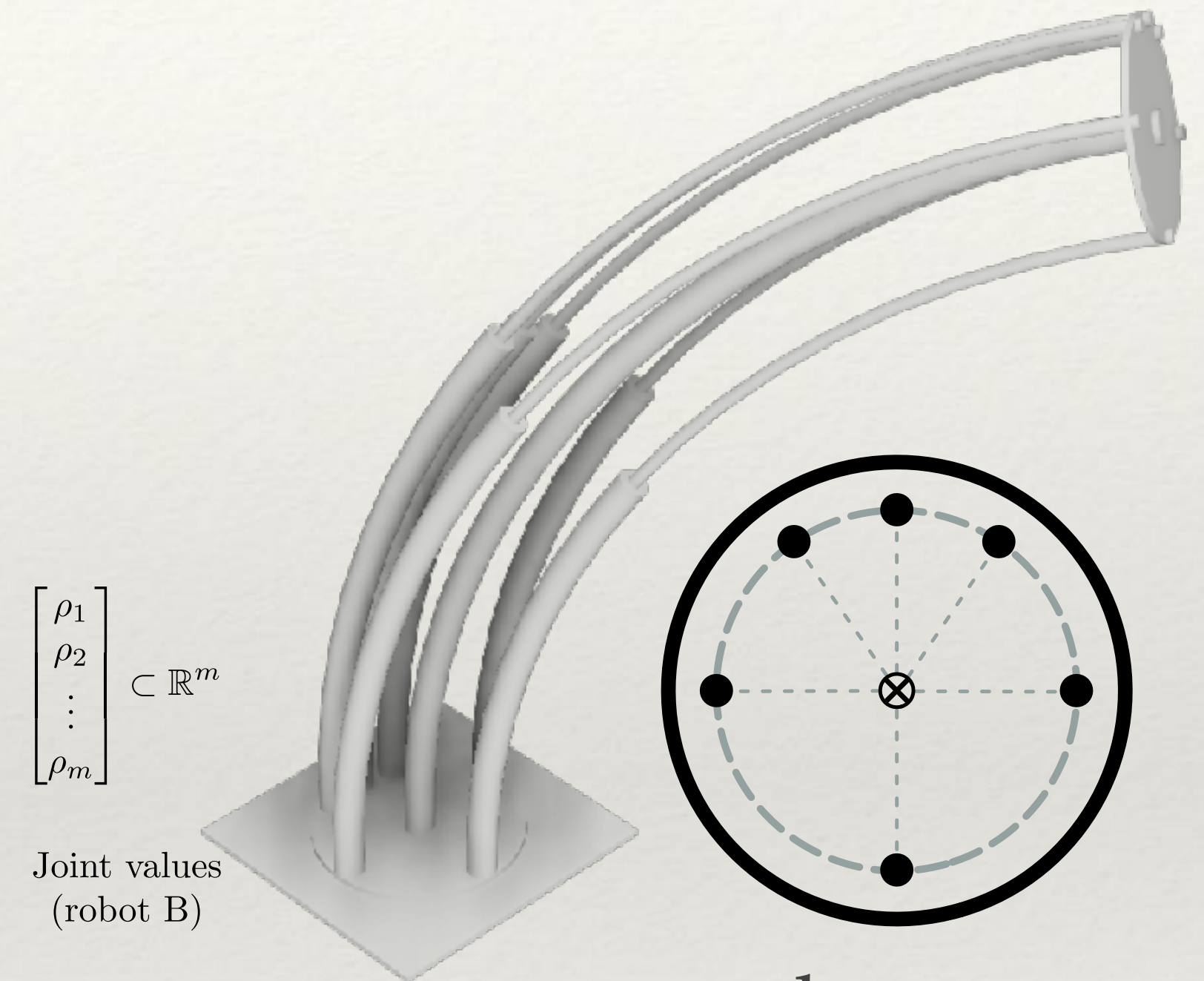


Encoder-Decoder Architecture



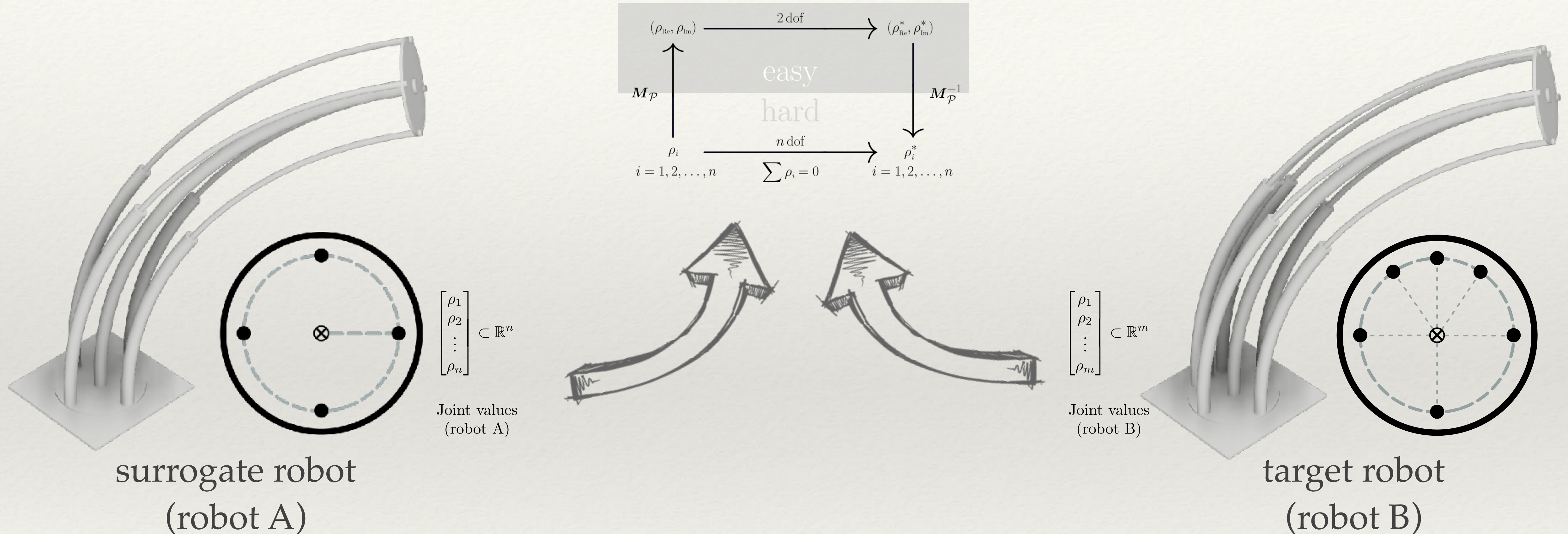
surrogate robot
(robot A)

*“Boundary condition:
Don’t reinvent the wheel”*

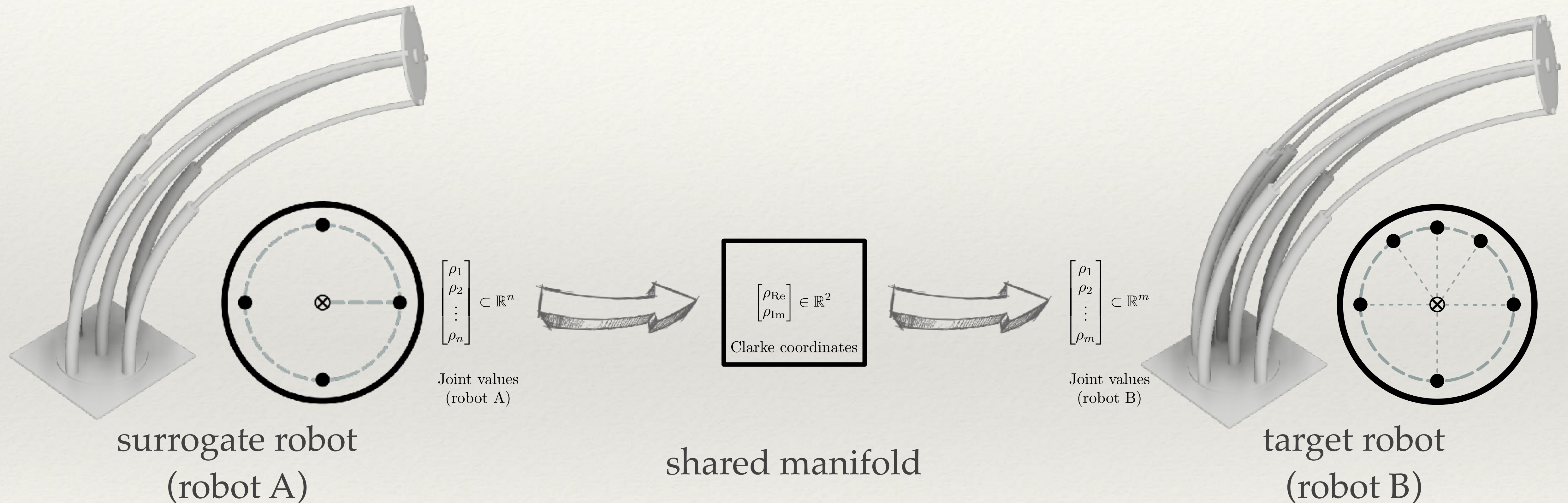


target robot
(robot B)

Encoder-Decoder Architecture

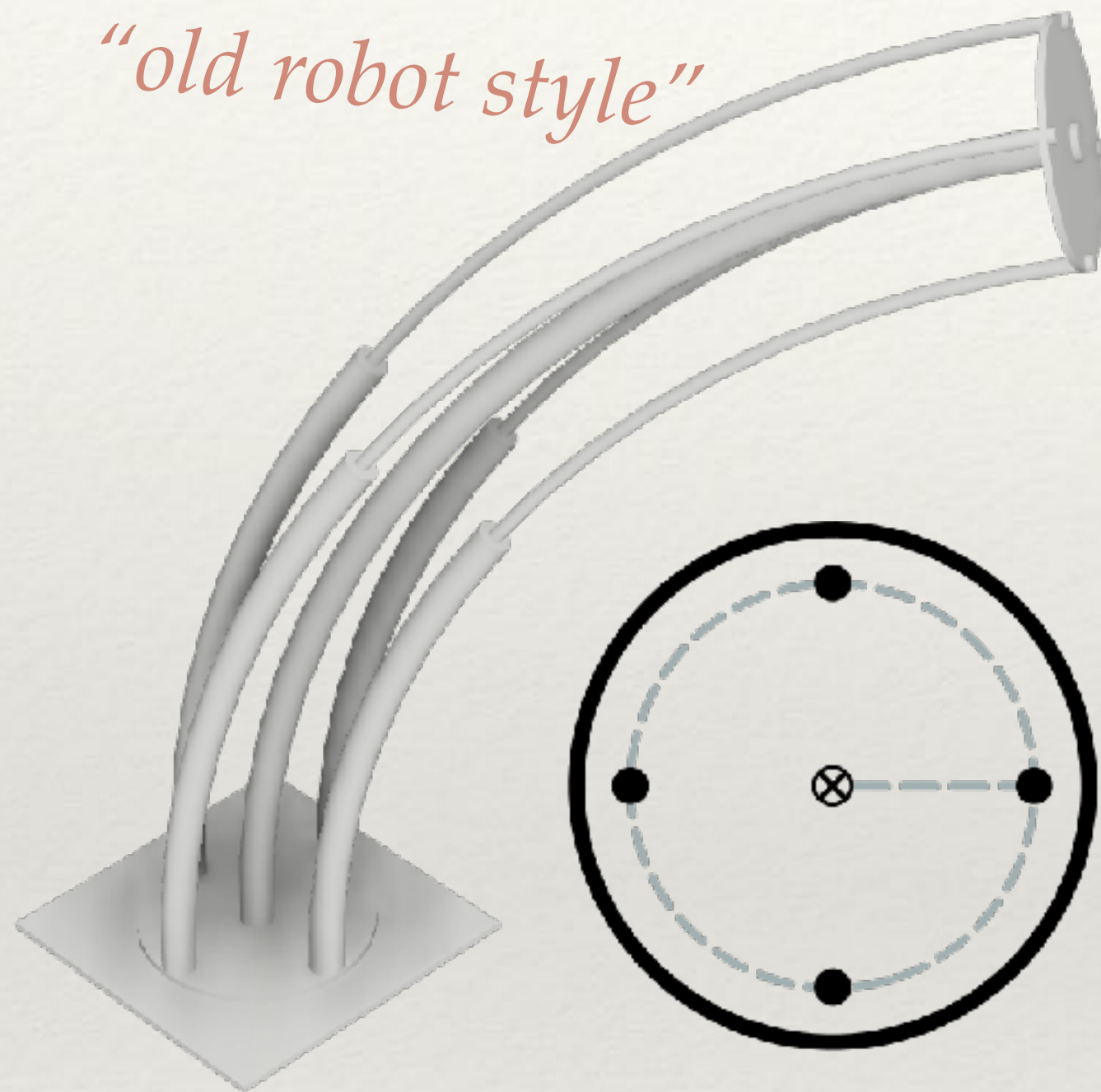


Encoder-Decoder Architecture



Encoder-Decoder Architecture

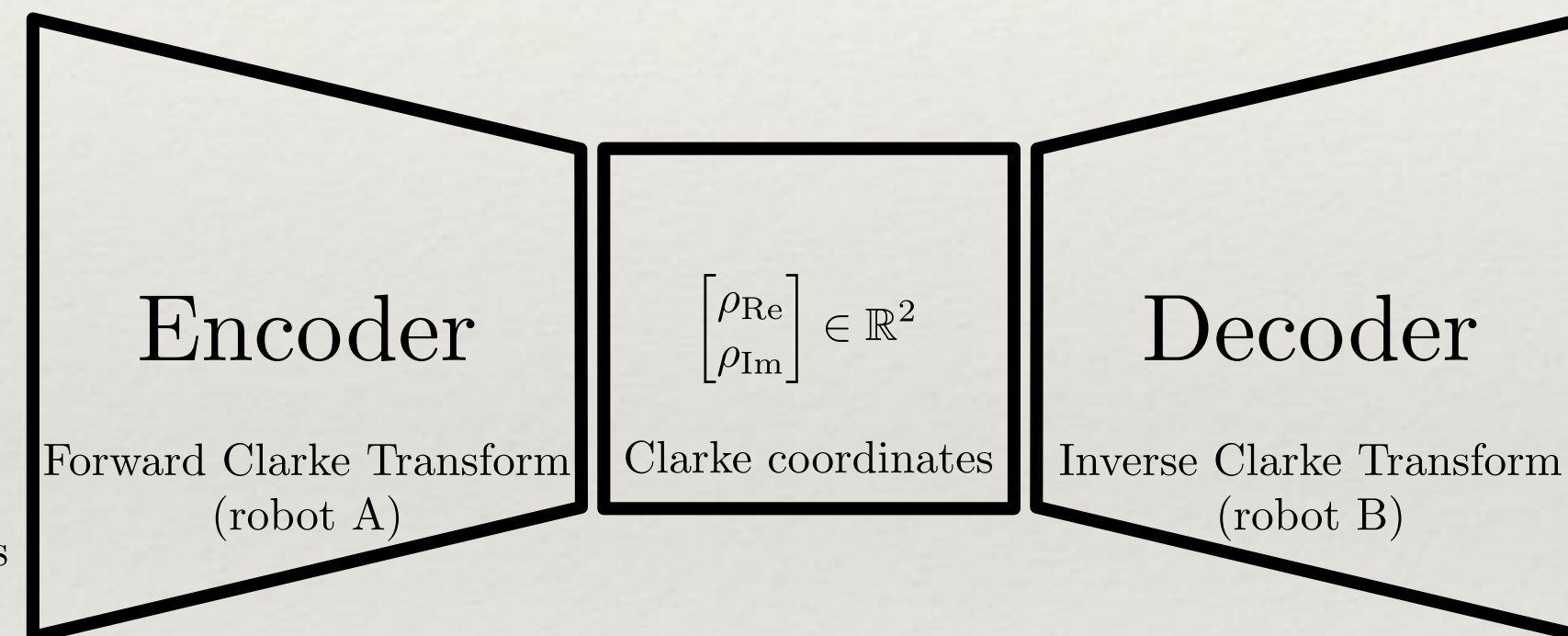
“old robot style”



surrogate robot
(robot A)

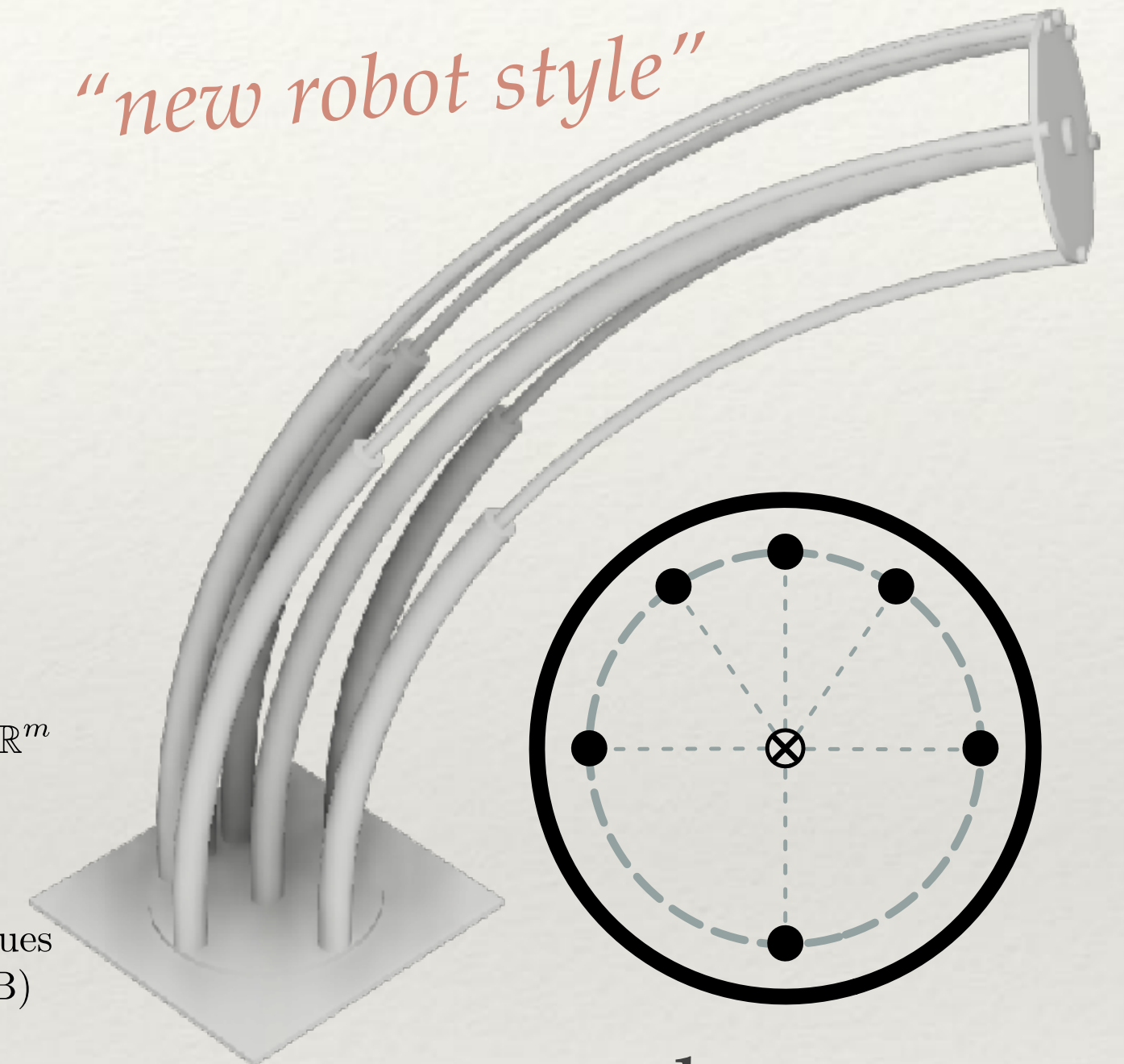
$$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{bmatrix} \in \mathbb{R}^n$$

Joint values
(robot A)



shared manifold

“new robot style”



target robot
(robot B)

$$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{bmatrix} \in \mathbb{R}^m$$

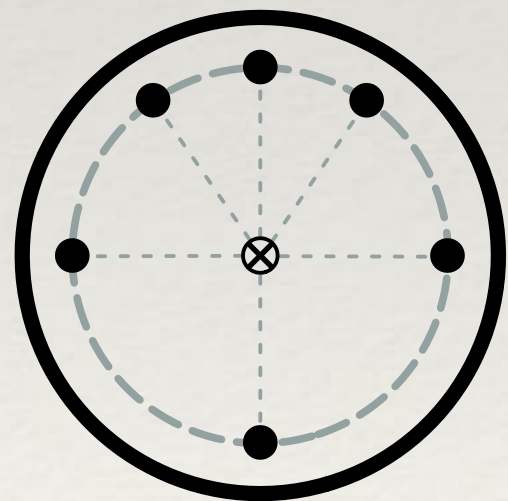
Joint values
(robot B)

“think of style transfer”

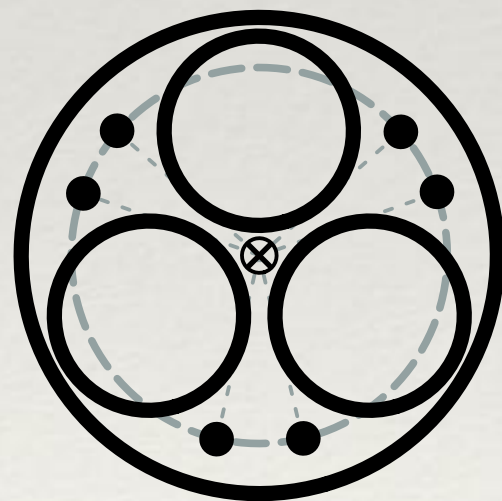
Closed-Form, Compact, and Interpretable

$$\boldsymbol{\rho}_{(\text{robot B})} = \overbrace{l_{(\text{robot B})} \text{diag} \left(d_{i,(\text{robot B})} \right) \mathbf{M}_{\mathcal{P}}^{-1}(\text{robot B})}^{\text{adds design parameters of robot B}} \cdot \underbrace{\frac{1}{l_{(\text{robot A})}} \mathbf{M}_{\mathcal{P}}(\text{robot A}) \text{diag} \left(\frac{1}{d_{i,(\text{robot A})}} \right)}_{\text{removes design parameters of robot A}} \boldsymbol{\rho}_{(\text{robot A})}$$

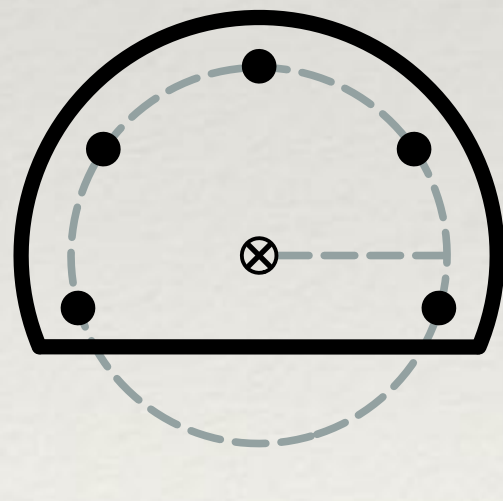
joint location



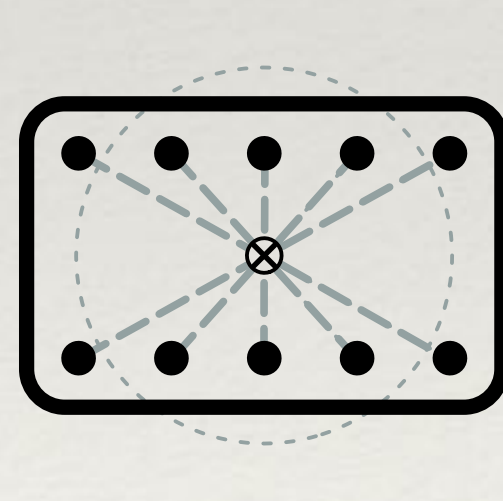
gravity reinforcement



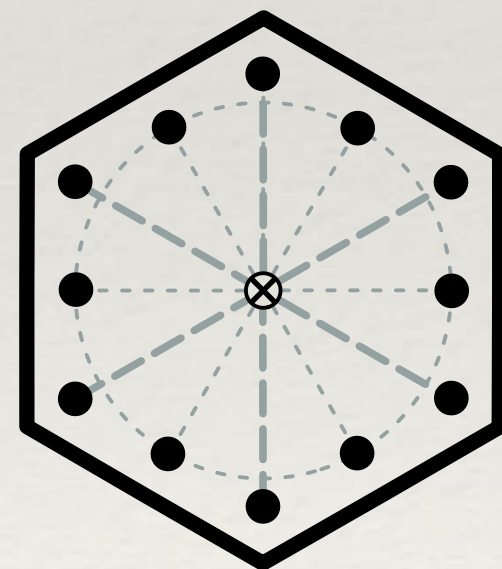
working channels



non-circular

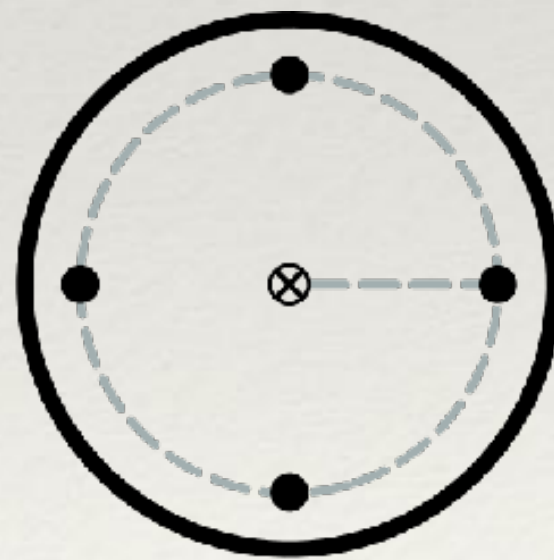
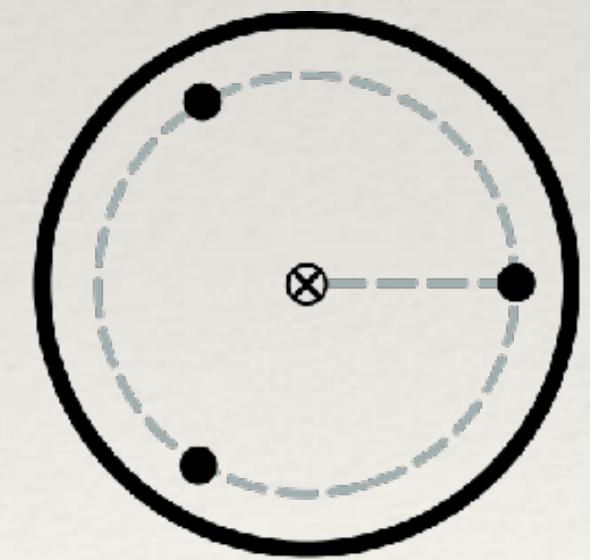


rectangle

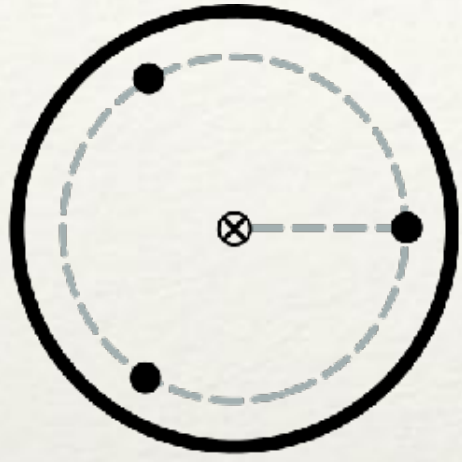
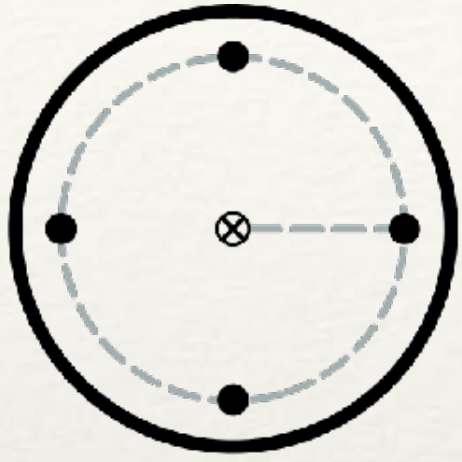
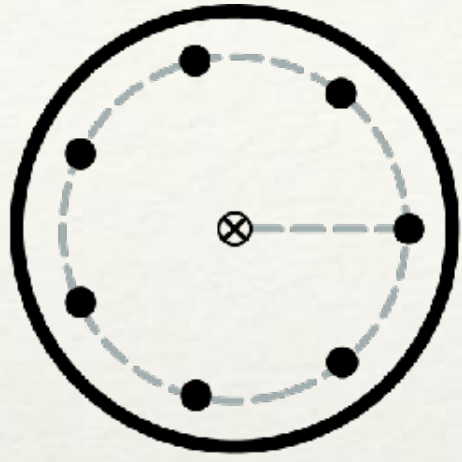
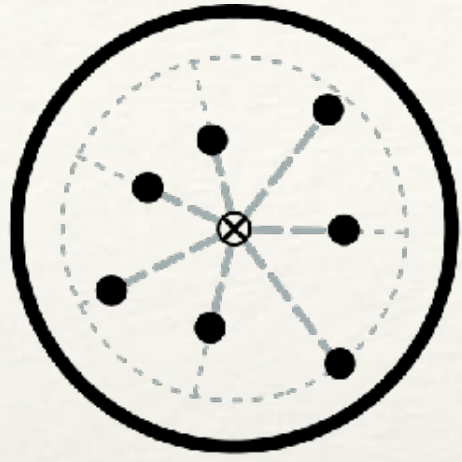
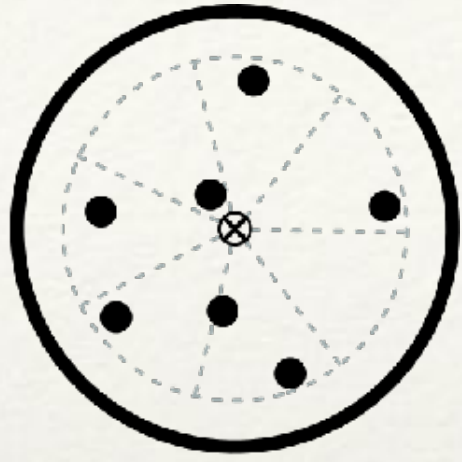


honeycomb

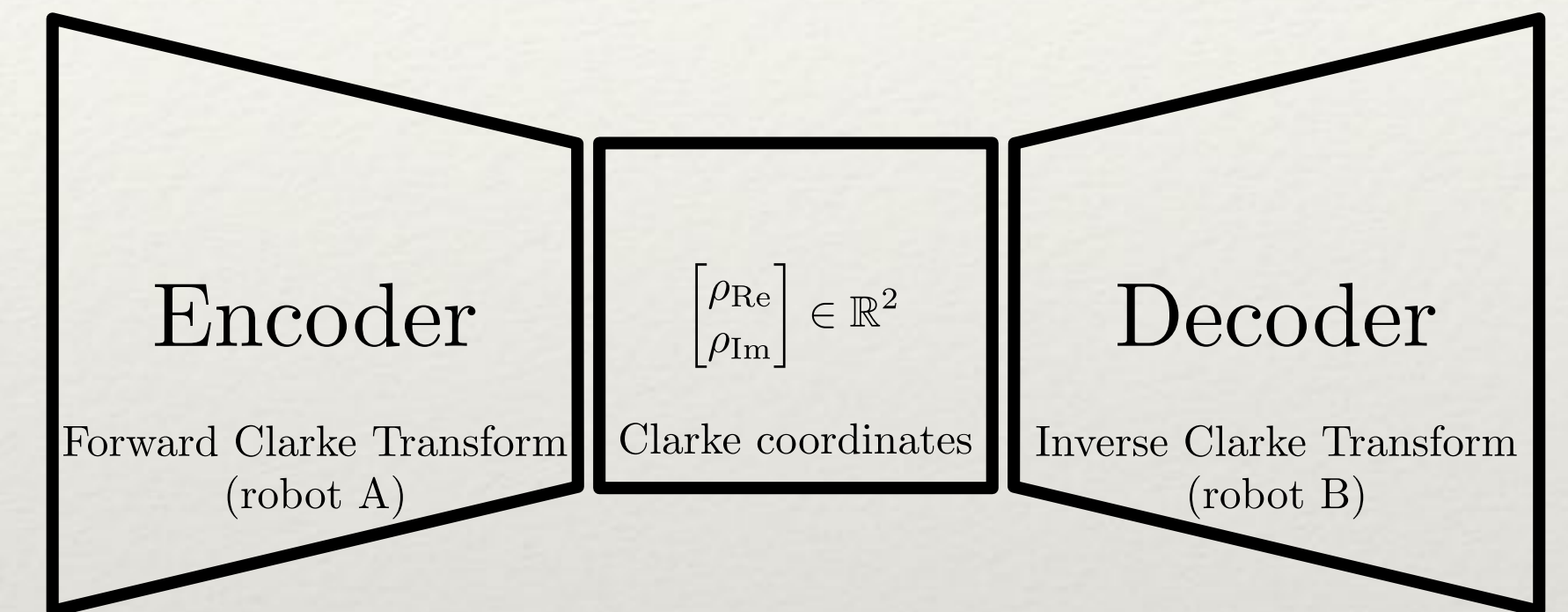
joint location



Contribution

joint location					
	Allen <i>et al.</i>	Della Santina <i>et al.</i>	Grassmann <i>et al.</i>	Grassmann <i>et al.</i>	Ours
arbitrary joint number n	✗	✗	✓	✓	✓
arbitrary distance d_i	✗	✗	✗	✓	✓
arbitrary angle ψ_i	✗	✗	✗	✗	✓

arbitrary joint locations for desired joint locations

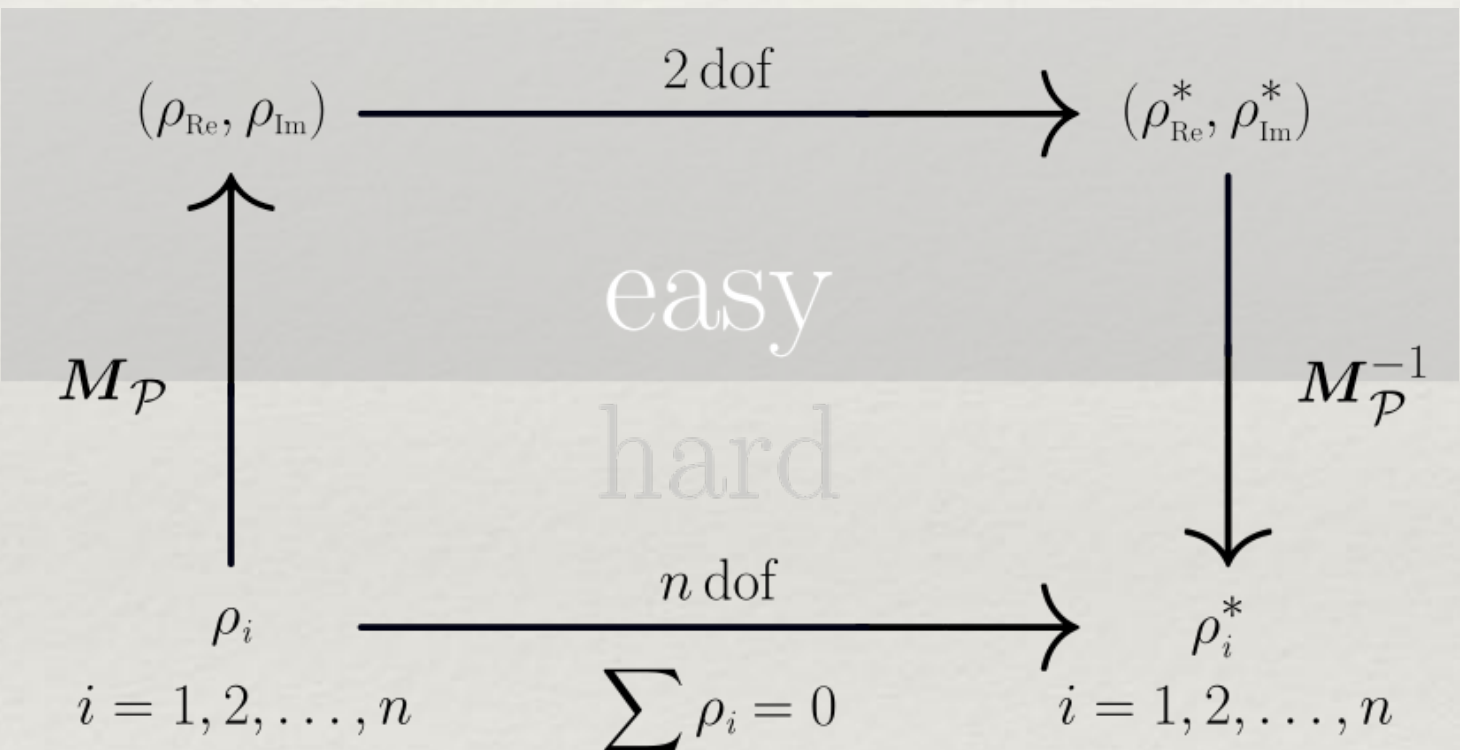


method for re-using previous methods

Thank you for your
time and attention



Paper ID 22



Call for Action:
Use Clarke Transform

Going beyond the limitations of three and four joints per segments. **Customize the joint location** to your needs and reuse all previous methods.

Session AA-4

joint location

gravity reinforcement working channels non-circular rectangle honeycomb

Paper ID 22

Clarke Transform and Encoder-Decoder Architecture for Arbitrary Joints Locations in Displacement-Actuated Continuum Robots

Reinhard M. Grassmann & Jessica Burgner-Kahrs

Motivation

- current joint configuration
 - limited to $n = 3$ and $n = 5$
 - symmetric
- desired joint configuration
 - task specific

Approach

- use Clarke Transform
- exploit shared 2-dof manifold
 - Clarke coordinates
 - map onto manifold, map back to joint space
- encoder-decoder architecture

Results

- arbitrary joint location
 - from any to specific
- form surrogate robot to specific robot
- solution is linear and exact

adds design parameters of robot B
removes design parameters of robot A

$$\rho_{(\text{robot B})} = l_{(\text{robot B})} \text{diag}(d_{t,(\text{robot B})}) M_P^{-1}(\text{robot B}) \left(\frac{1}{l_{(\text{robot A})}} M_P(\text{robot A}) \text{diag}\left(\frac{1}{d_{t,(\text{robot A})}}\right) \rho_{(\text{robot A})} \right)$$

Conclusion

- overcome limitation
- new possibilities
 - algorithmic
 - mechanical design

encoder-decoder architecture

surrogate robot (robot A) target robot (robot B)

Clarke Transform and Clarke coordinates

Controller

uncertainty of a joint location