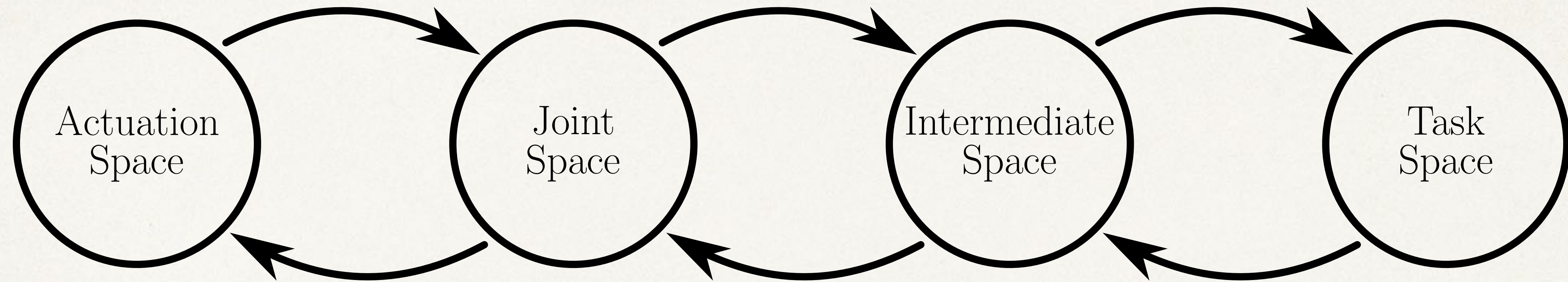


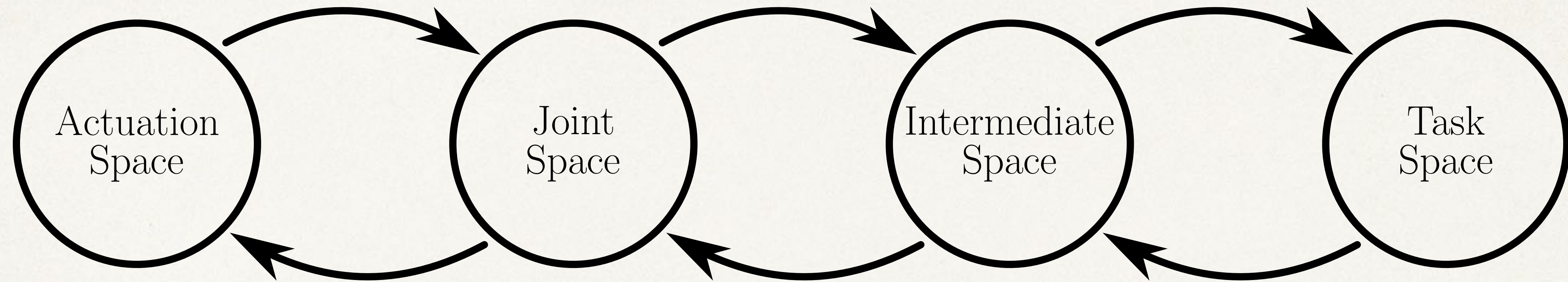
Clarke Transform: *A Lingua Franca* for Continuum Robotics

Benchmarking in Soft Robotics: Standardizing Data Collection and Evaluation for Actuation, Sensing, and Control

Reinhard M. Grassmann

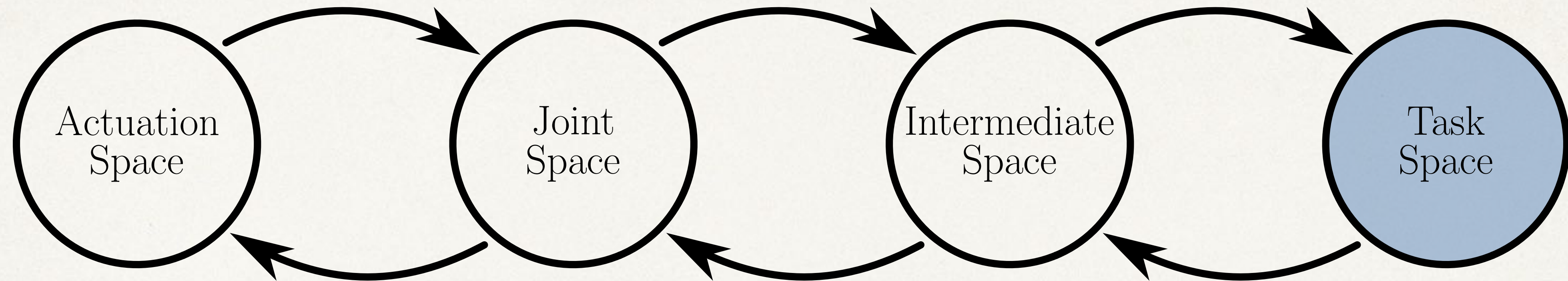


Robotics is the art of transformations between spaces

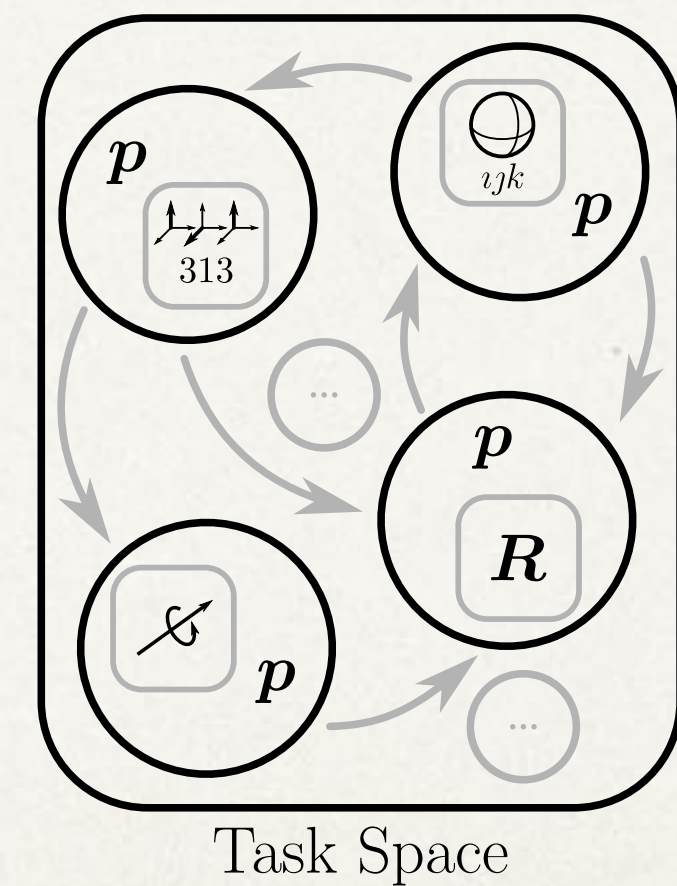


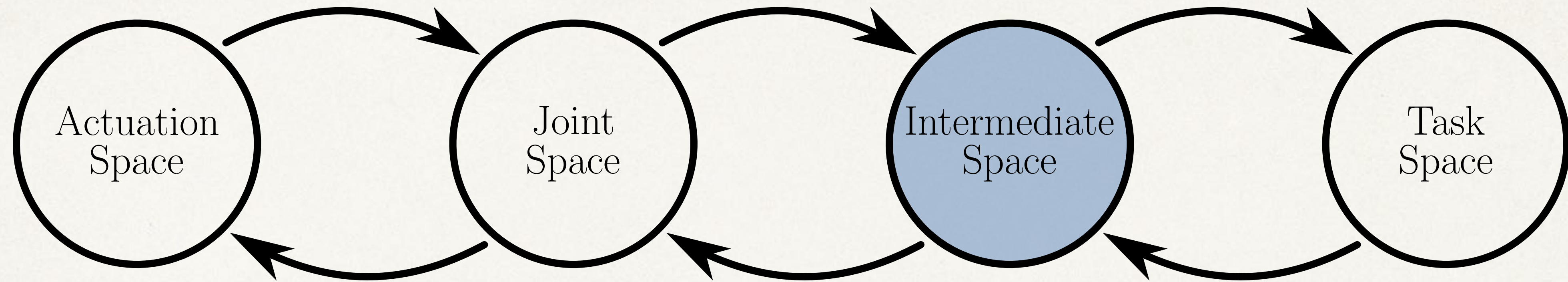
Robotics is the art of transformations between spaces

At the same time, each of the spaces has its own pitfalls.

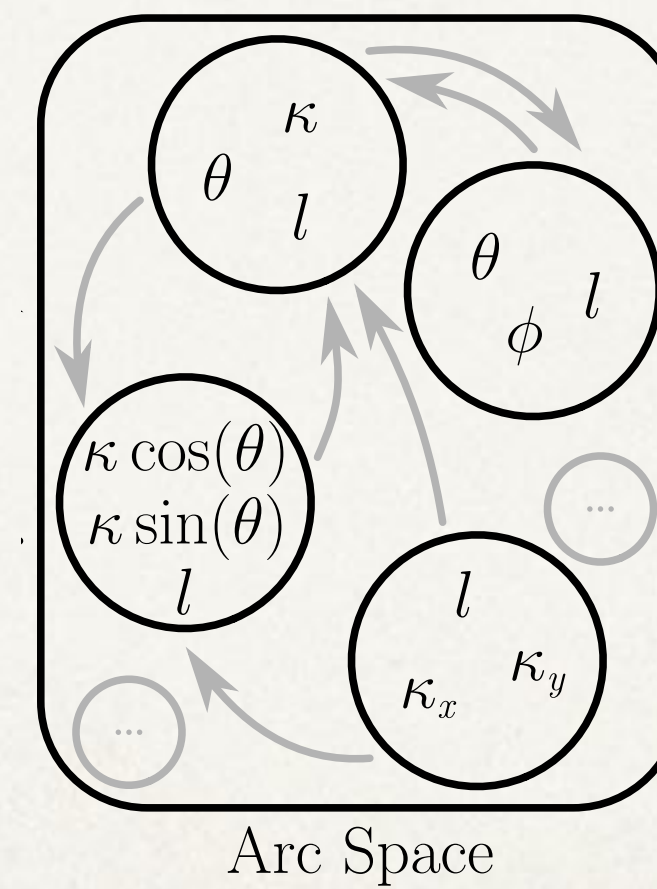


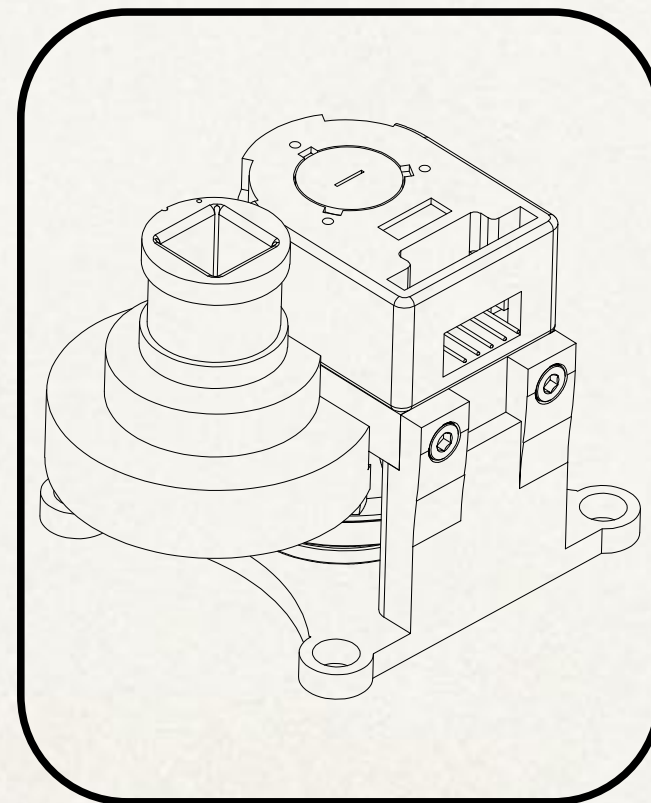
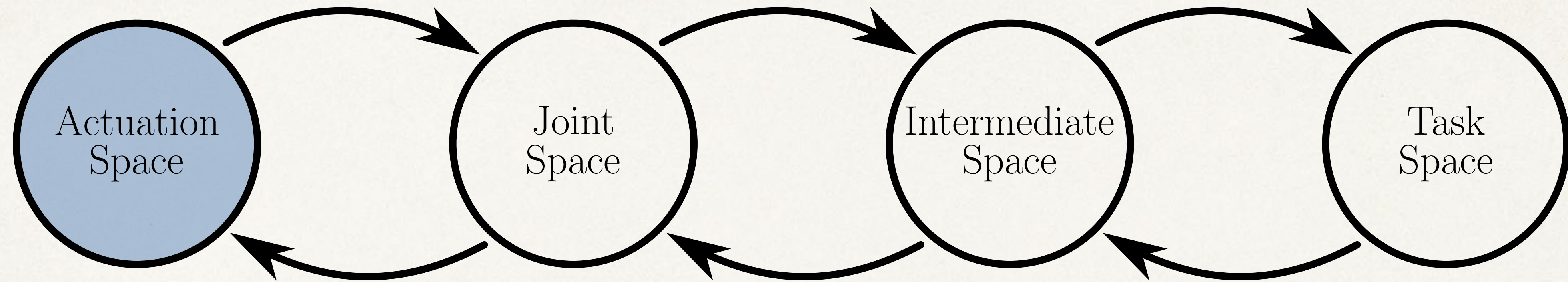
- ❖ Euler angles
 - ❖ 12 sets, singularities, ...
- ❖ unit quaternions
 - ❖ double coverage, unit length
- ❖ rotation matrix
 - ❖ 9 values, 6 constraints





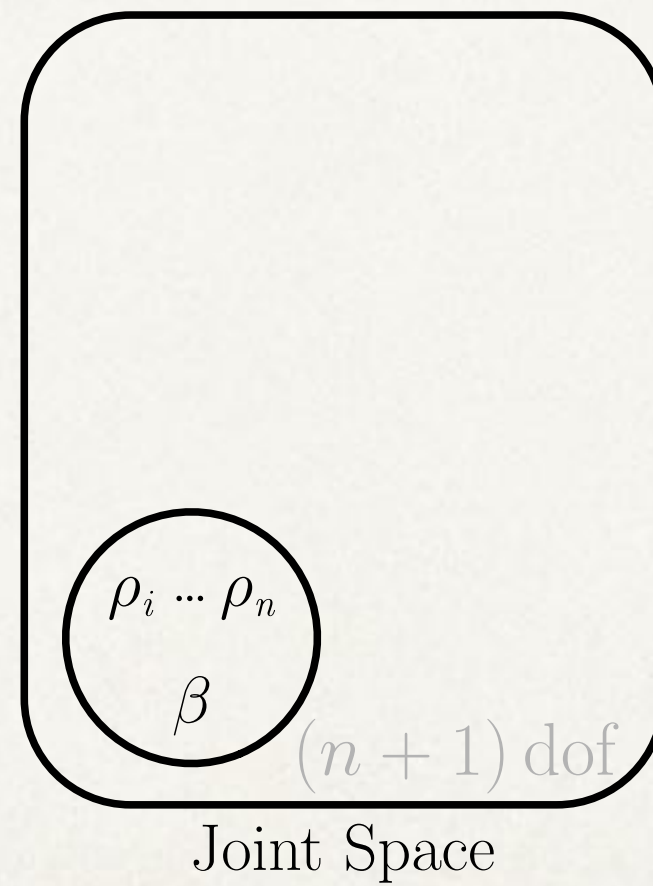
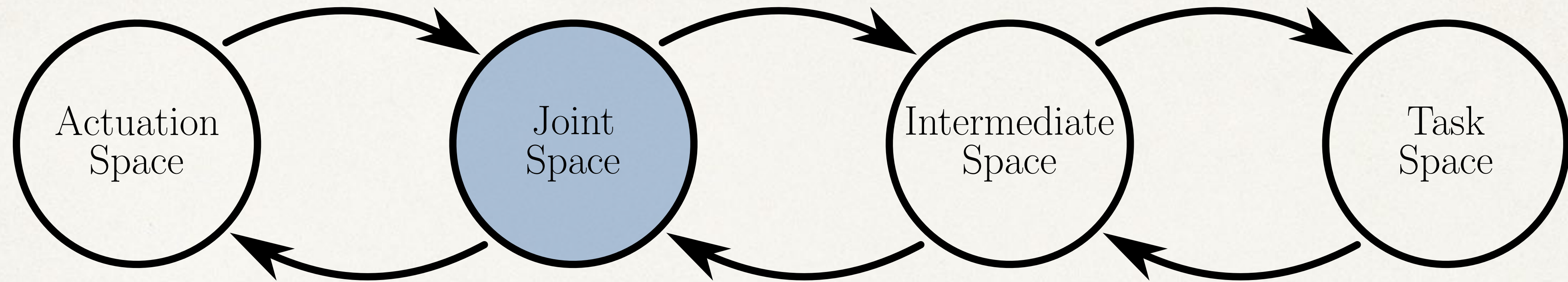
- ❖ curvature
- ❖ coordinate singularities, ...
- ❖ arc parameters
 - ❖ linear combination
 - ❖ non-linear combination
- ❖ intermediate space





Actuation Space

- ❖ flow and effort of the system
- ❖ mostly position controlled
- ❖ a few torque controlled

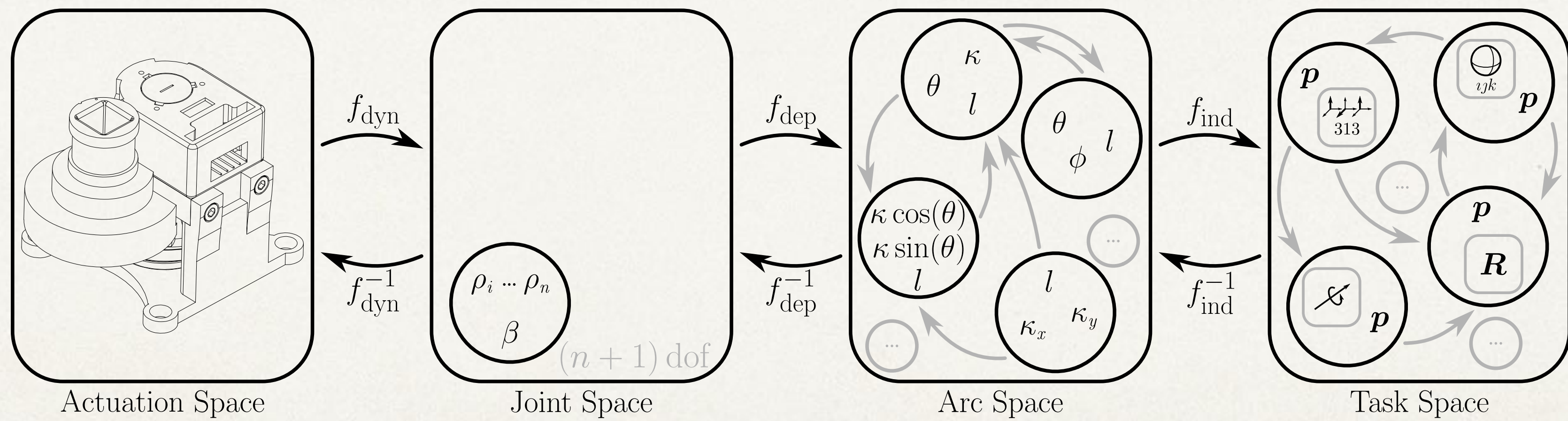
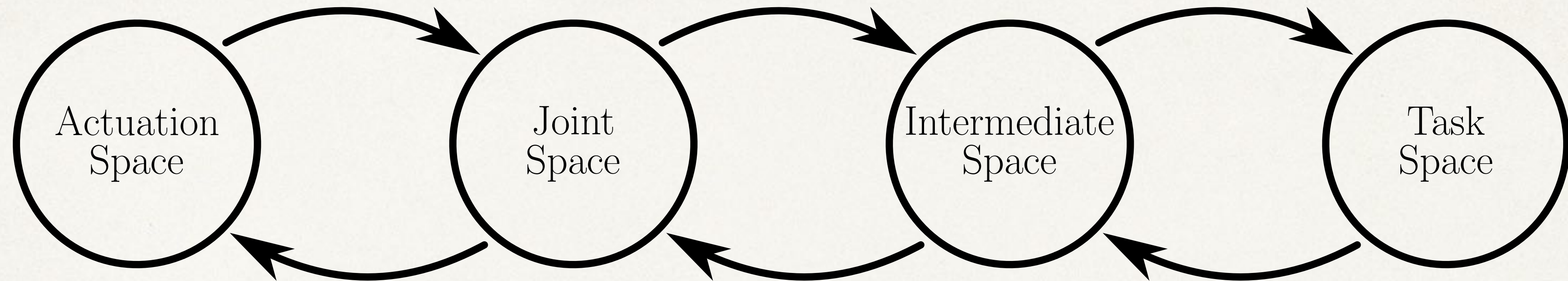


- ❖ improved state representations
- ❖ limited to 3 and 4 joints
- ❖ missing manifold
- ❖ overlooked constraints

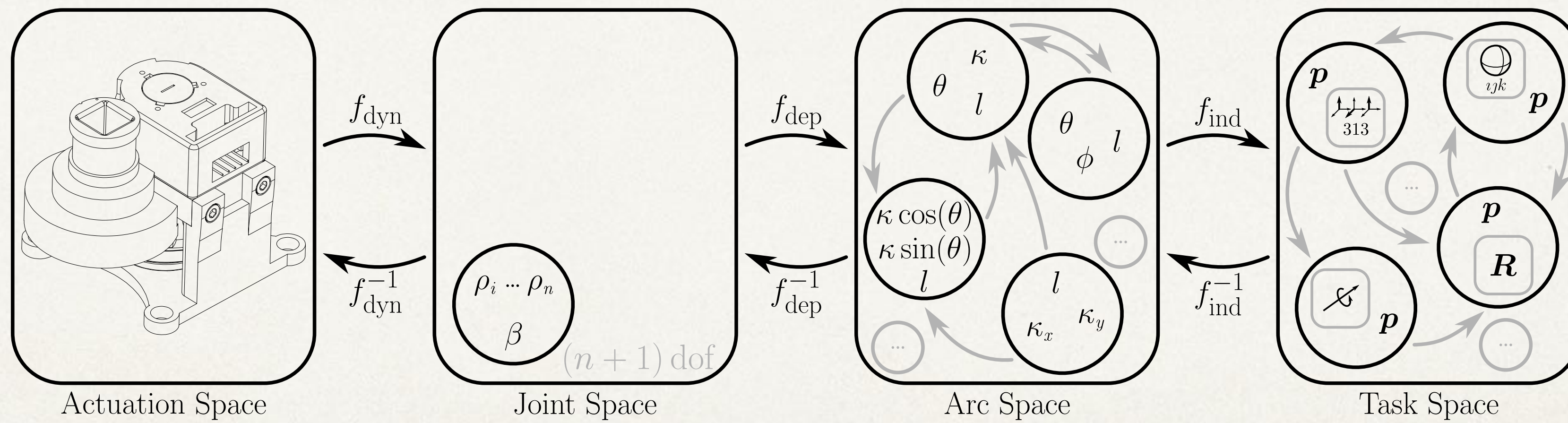
[Della Santina et al., RA-L 2020] “On an Improved State Parametrization for Soft Robots with Piecewise Constant Curvature and Its Use in Model-Based Control”

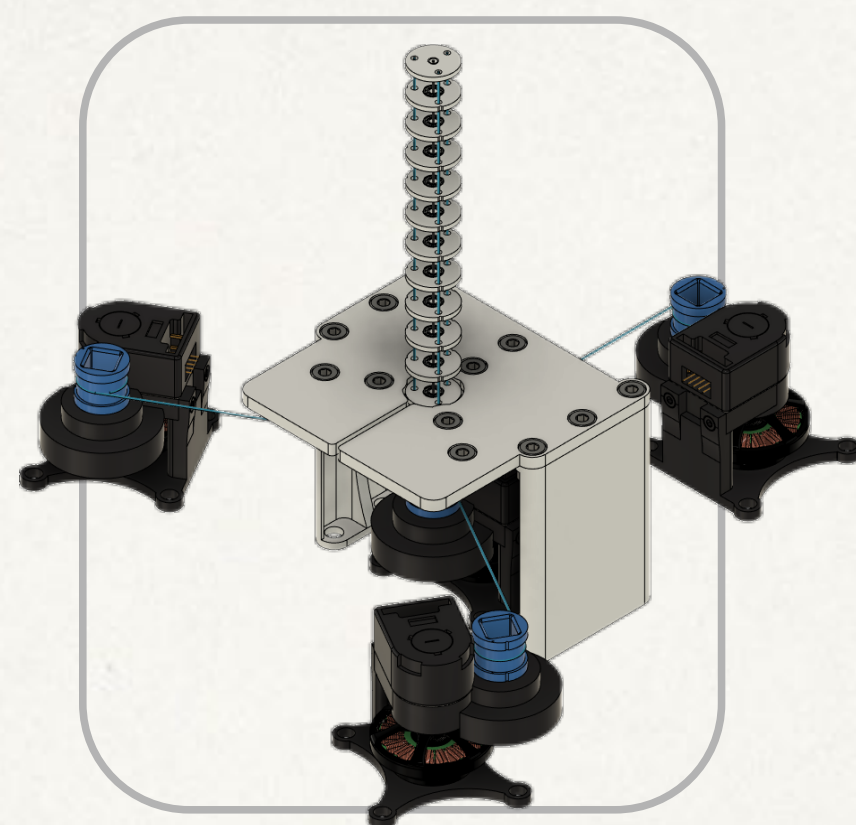
[Allen et al., RoboSoft 2020] “Closed-Form Non-Singular Constant-Curvature Continuum Manipulator Kinematics”

[Dian et al., Access 2022] “A Novel Disturbance-Rejection Control Framework for Cable-Driven Continuum Robots With Improved State Parameterization”

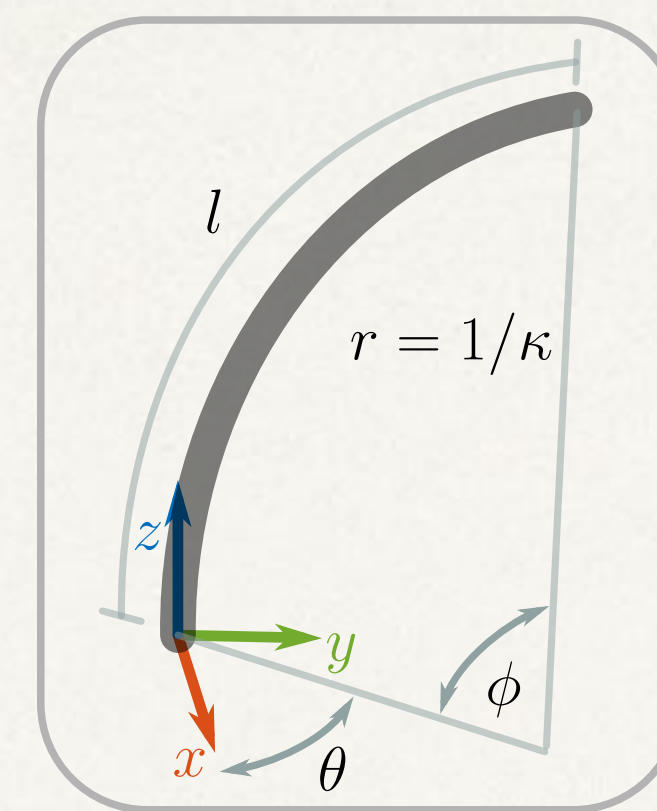


We need to master this

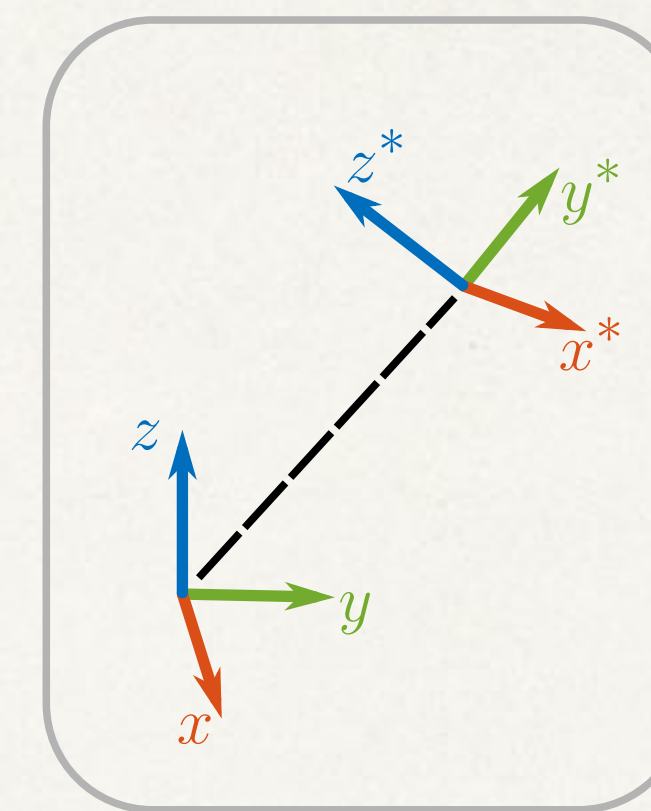




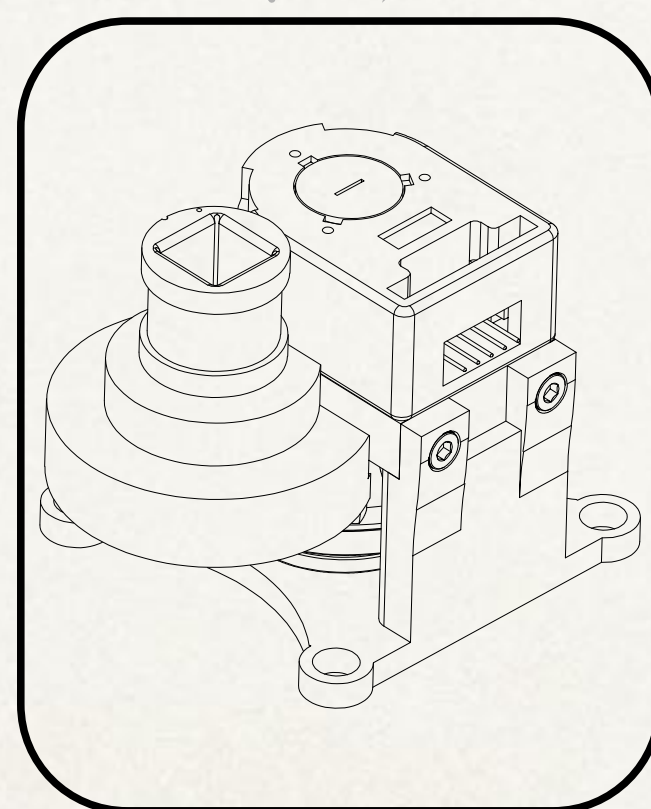
Physical Hardware



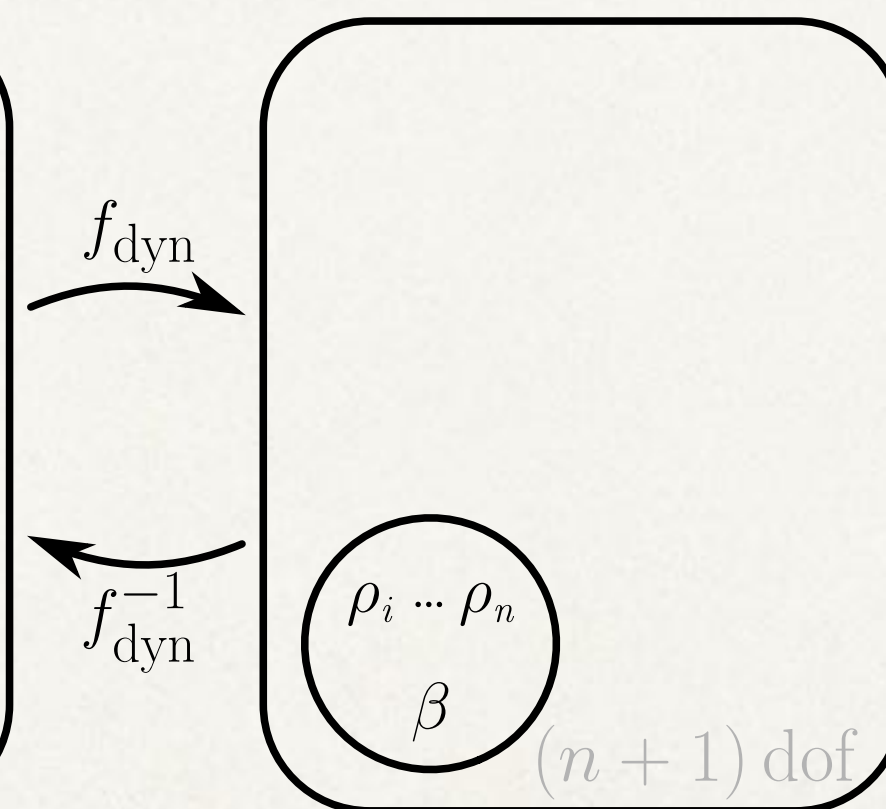
Constant Curvature Assumption



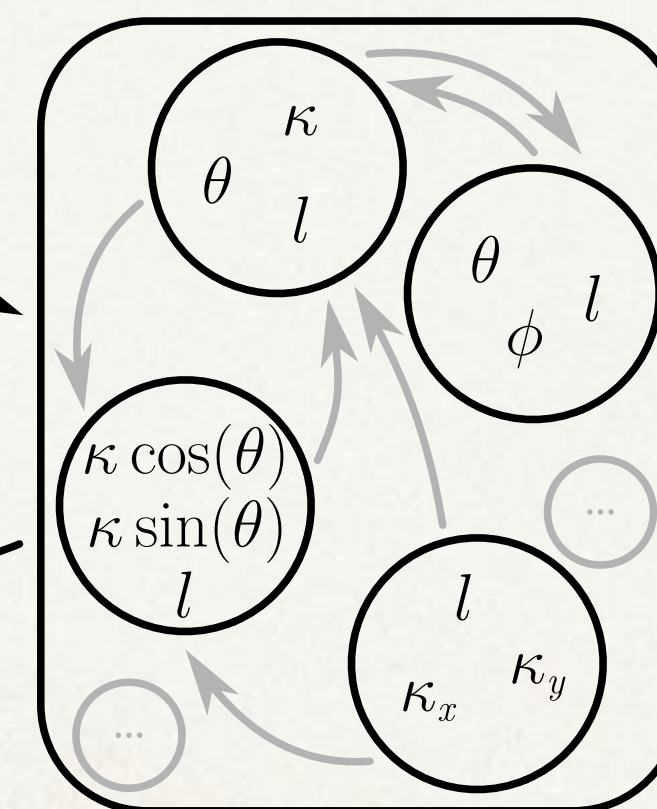
Rigid Transformation



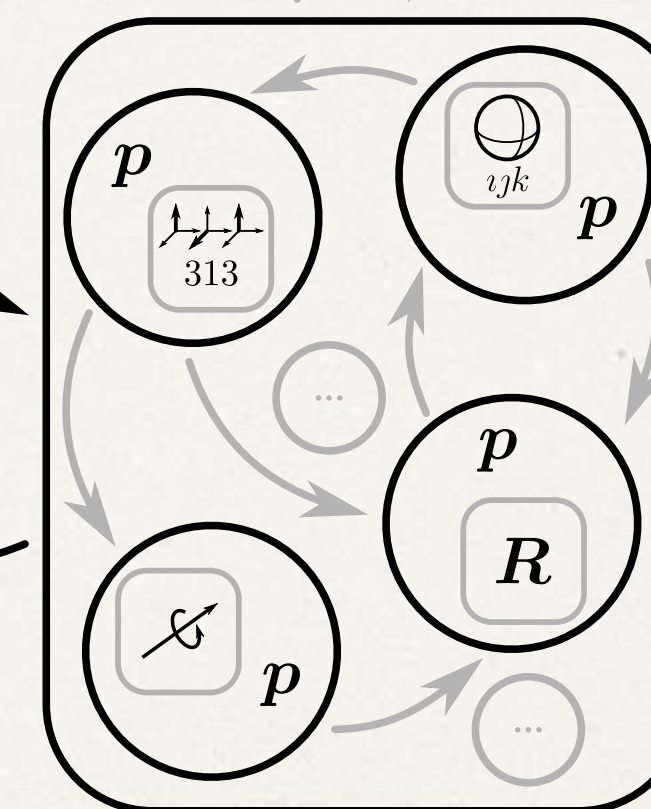
Actuation Space



Joint Space



Arc Space



Task Space

f_{dyn}

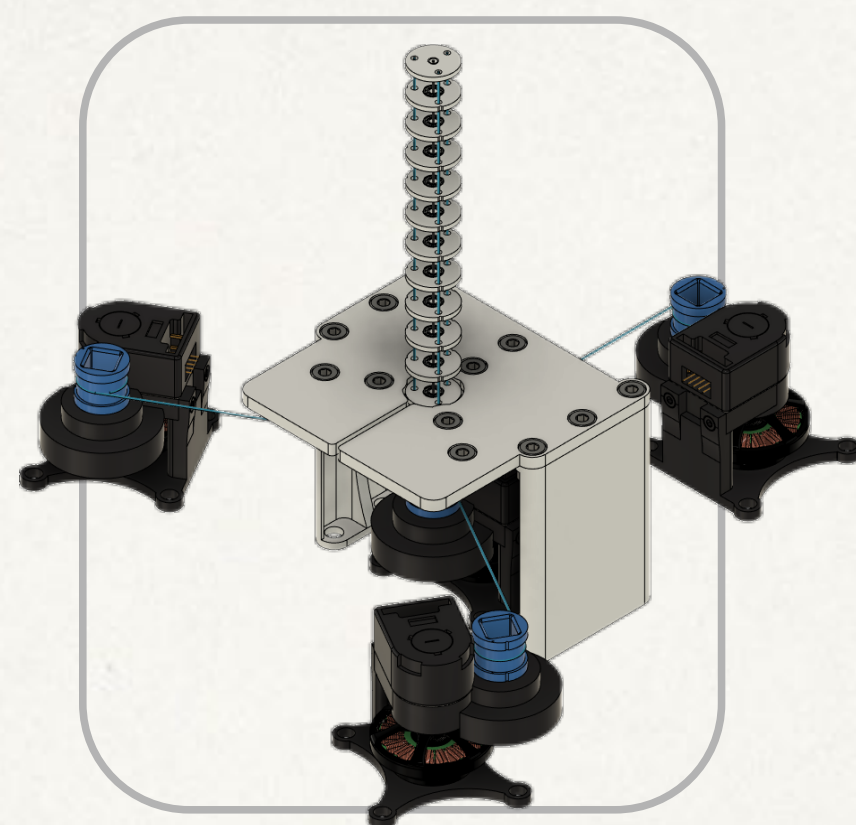
f_{dyn}^{-1}

f_{dep}

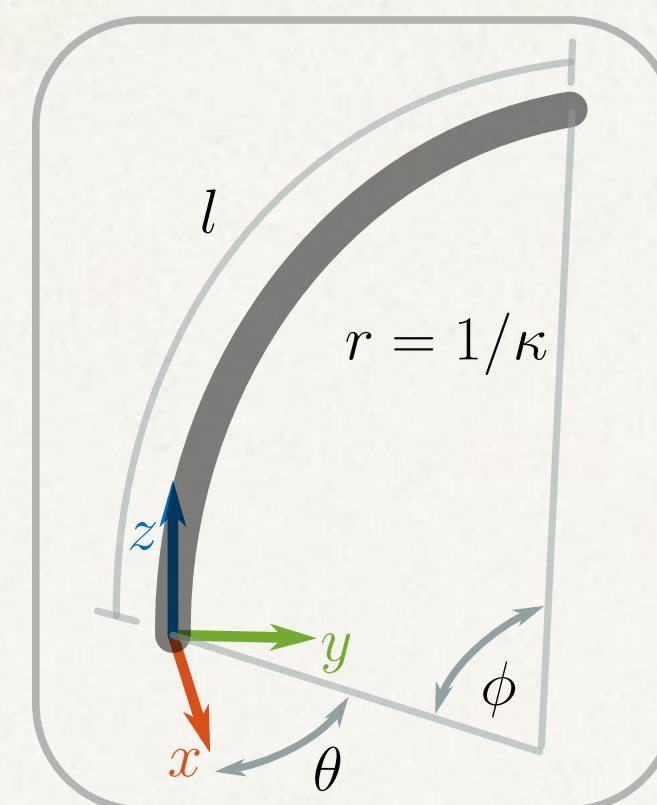
f_{dep}^{-1}

f_{ind}

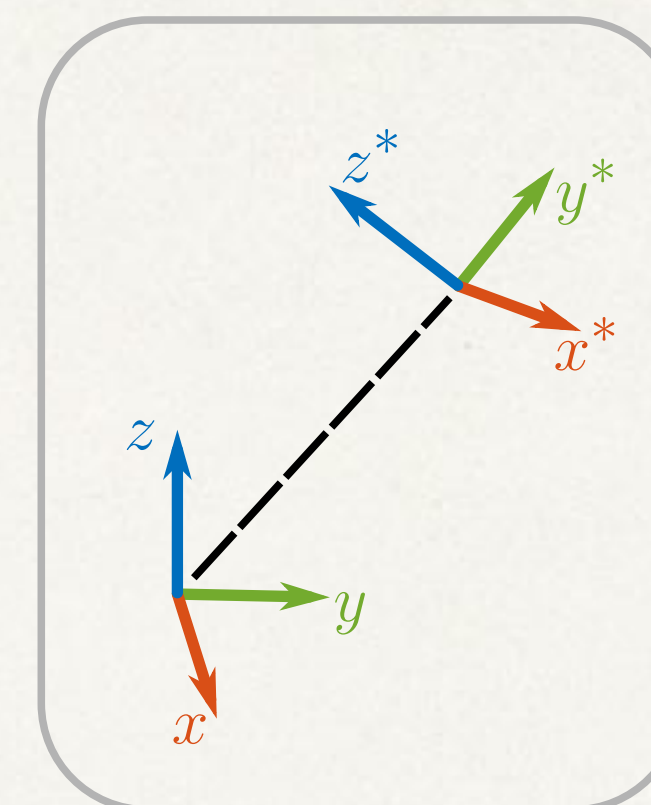
f_{ind}^{-1}



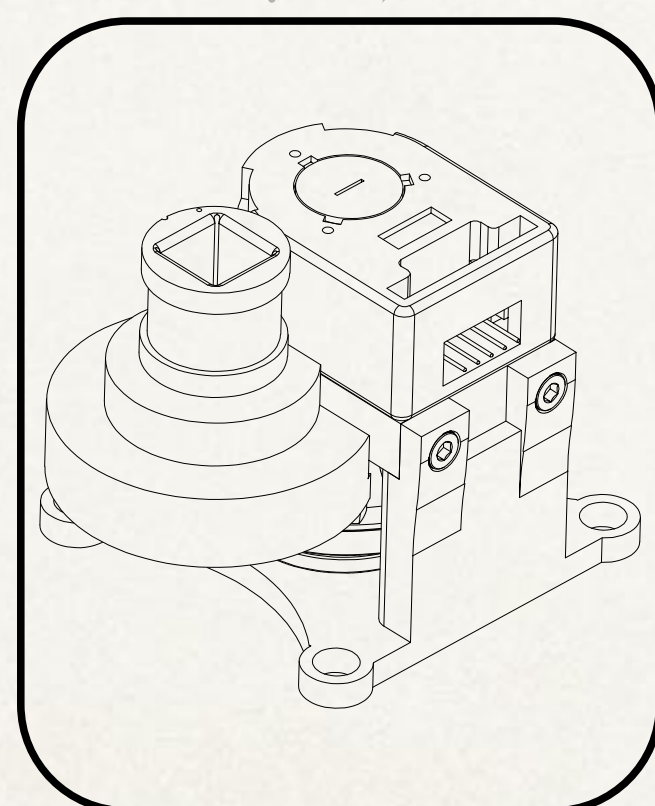
Physical Hardware



Constant Curvature Assumption



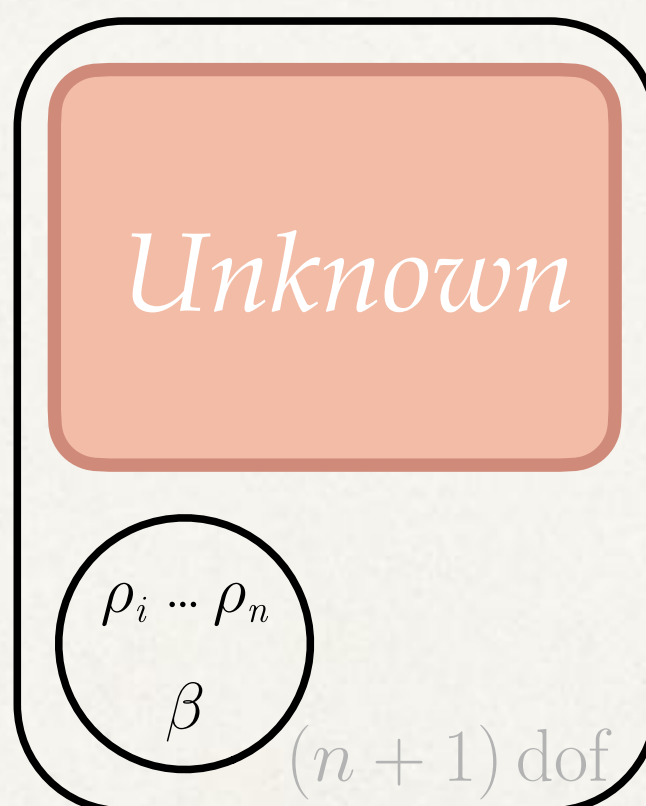
Rigid Transformation



Actuation Space

f_{dyn}

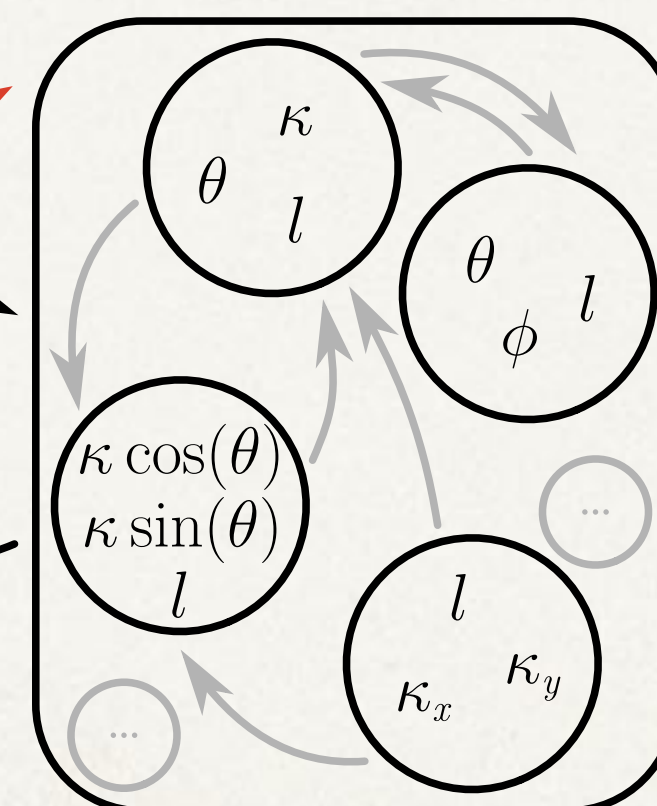
f_{dyn}^{-1}



Joint Space

f_{dep}

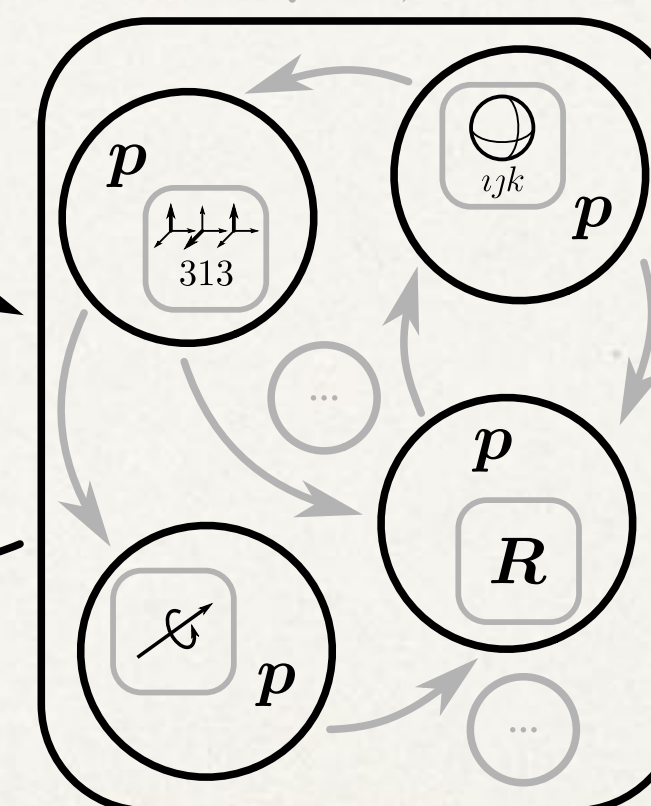
f_{dep}^{-1}



Arc Space

f_{ind}

f_{ind}^{-1}



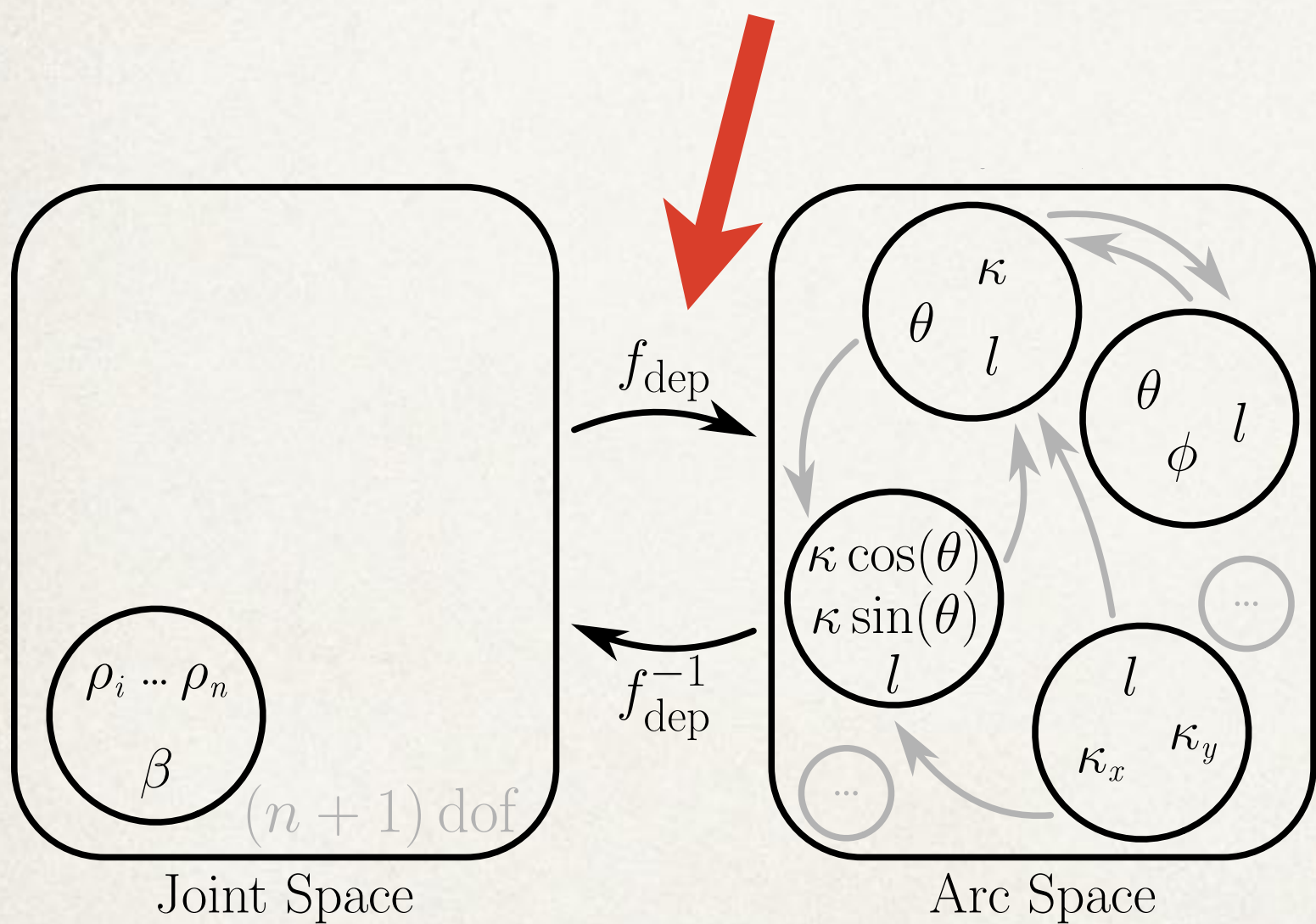
Task Space

Ripple Effects



known for $n=3$ and $n=4$ actuators

robot-dependent mapping for n actuators ?



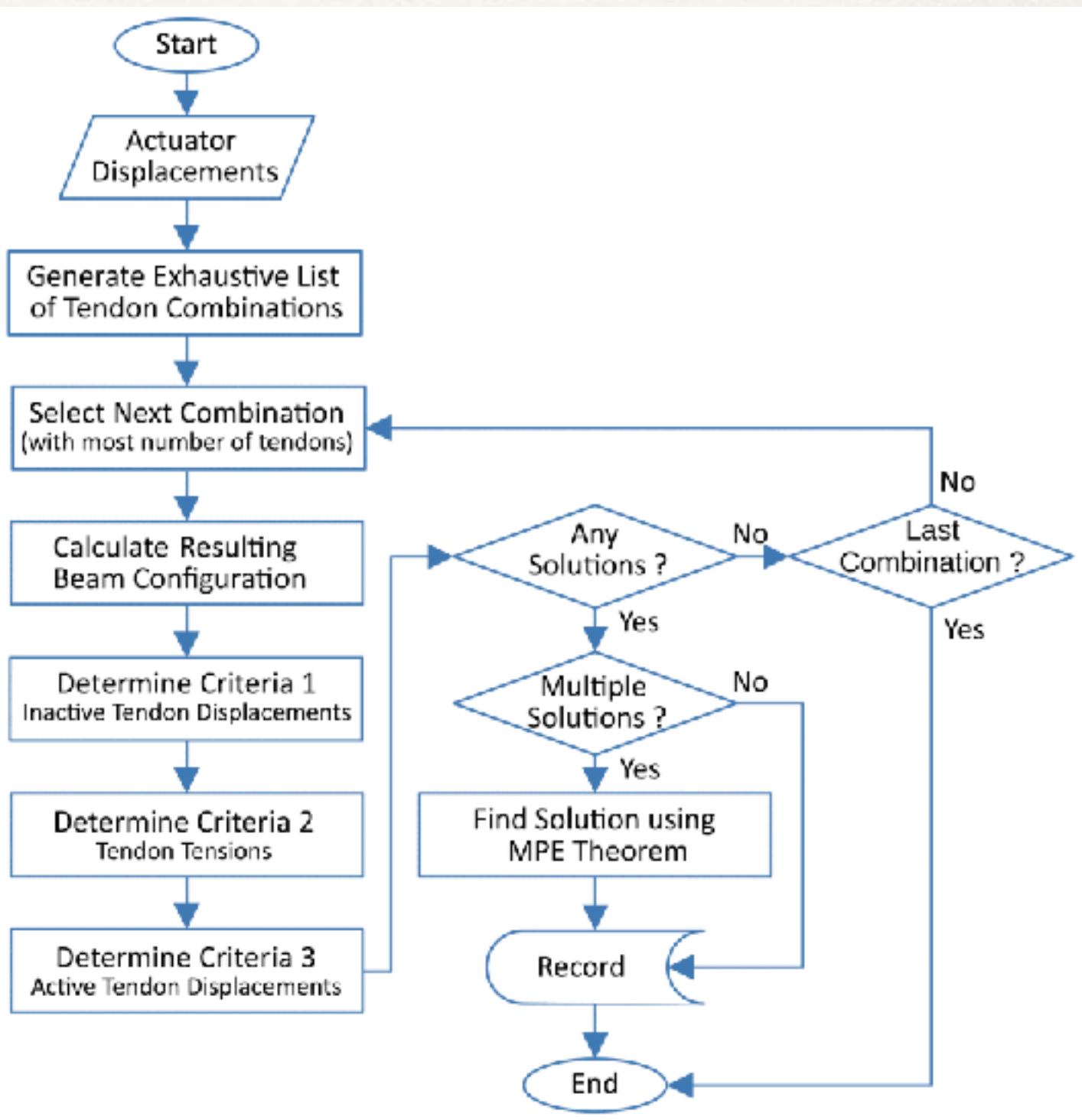
the problem

```
input : Actuator displacements ([ΔLact]), tendon locations ([αact], [Ract]) and geometry and mechanical properties of beam and tendons
output: Active/Slack set of tendons and the resulting beam configuration ([θ, φ, Lc])

Algorithm SolveForwardKinematics ([ΔLact], [αact], [Ract], [properties])
n ← size([αact], 1);
Generate exhaustive list of n- to 1-tendon combinations;
foreach j ∈ n do
  foreach i ∈ number of j-tendon combinations do
    θ0 ← 0;
    φ0 ← 0;
    Lc0 ← Lc;
    [comb] ← i-th j-tendon combinations;
    [θ, φ, Lc] = FindBeamConfig ([comb], [θ0, φ0, Lc0], [ΔLact], [αact], [Ract]);
    Find resulting tendon displacements [ΔLt]res using Equation (2);
    S1 ← Any([ΔLt]res out of [comb] larger than the corresponding [ΔLt]);
    Find resulting [comb] tendon tensions [F]t using Equation (3);
    S2 ← Not(Any([F]t < 0));
    Find resulting [comb] tendon stretches [δ]t using Equation (4);
    Find resulting [comb] actuator displacements using [ΔLact]res = [ΔLt]res + [δ]t;  
S3 ← Any([ΔLact]res of [comb] differs from the corresponding [ΔLact]);
    Compute the beam potential energy using Equation (5);
    Record status parameters of S1, S2, and S3, as well as the potential energy of this combination;
  [solindex], [solcomb] ← Find(combinations with all S1, S2, and S3 equal to 1);
  if size([solindex], 1) = 1 then
    solution ← [solcomb];
    return solution and [θ, φ, Lc];
  else if size([solindex], 1) > 1 then
    solution ← Find([solcomb] element with the least amount of potential energy);
    return solution and [θ, φ, Lc];
  else
    Proceed with the next list of (j - 1)-tendon combinations;
  end
end
return

Procedure FindBeamConfig ([comb], [θ0, φ0, Lc0], [ΔLact], [αact])
Set [ΔLact] based on [comb] and [ΔLact];
Set [αt] based on [comb] and [αact];
Set [Rt] based on [comb] and [Ract];
Derive tendon displacements [ΔLt] using Equation (2);
Derive tendon tensions [F]t using Equation (3);
Derive tendon stretches [δ]t using Equation (4);
Define equations [Eqi] = [ΔLact] - [ΔLt] - [δ]t;
Find beam configuration [θ, φ, Lc] by numerically solving the set of Equations [Eqi] = 0 based on the initial condition [θ0, φ0, Lc0];
return [θ, φ, Lc];
```

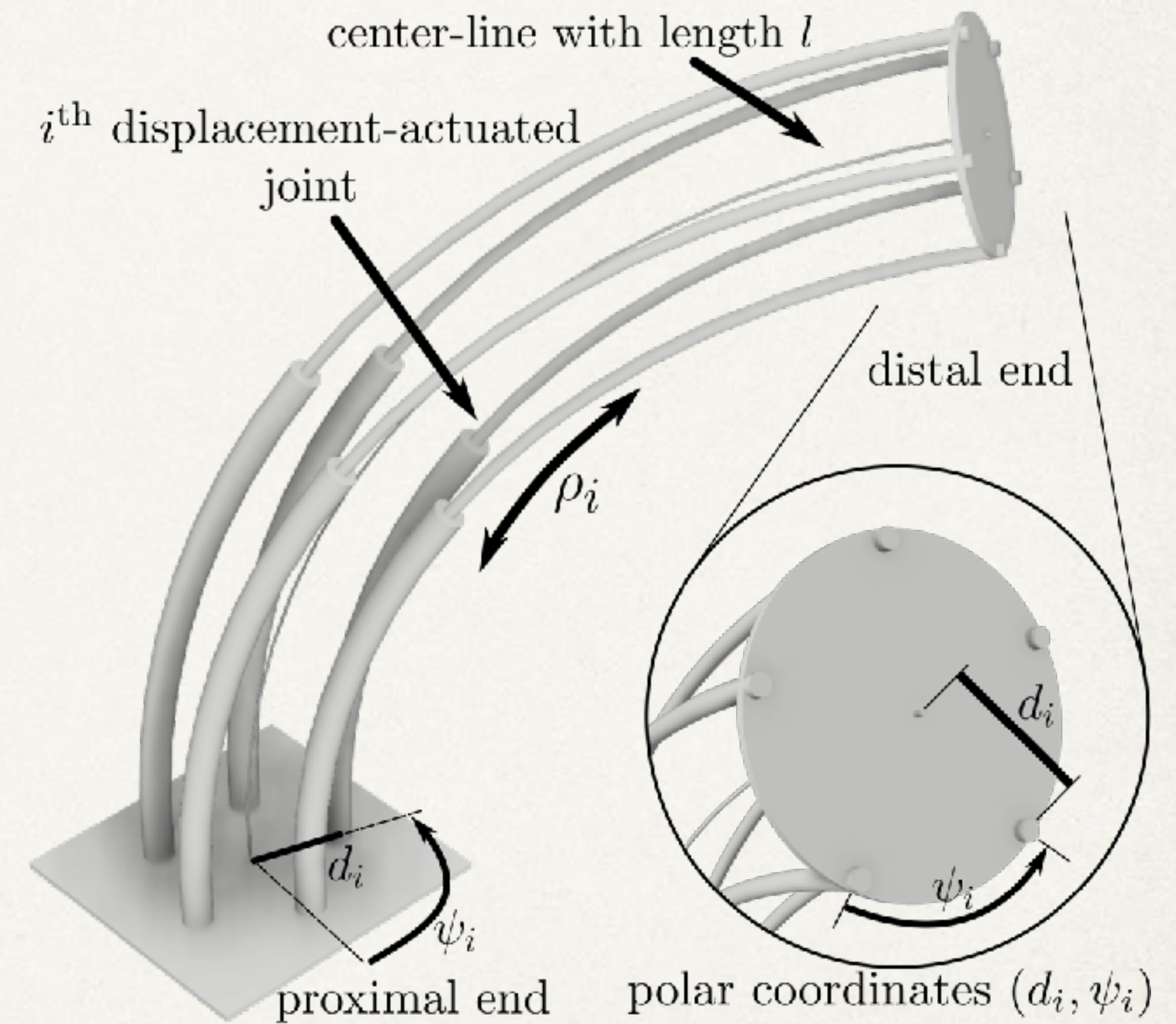
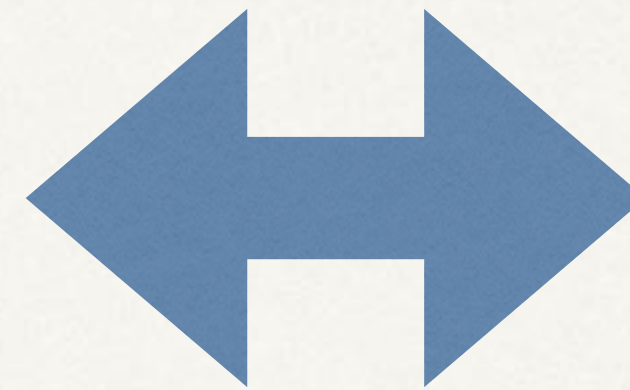
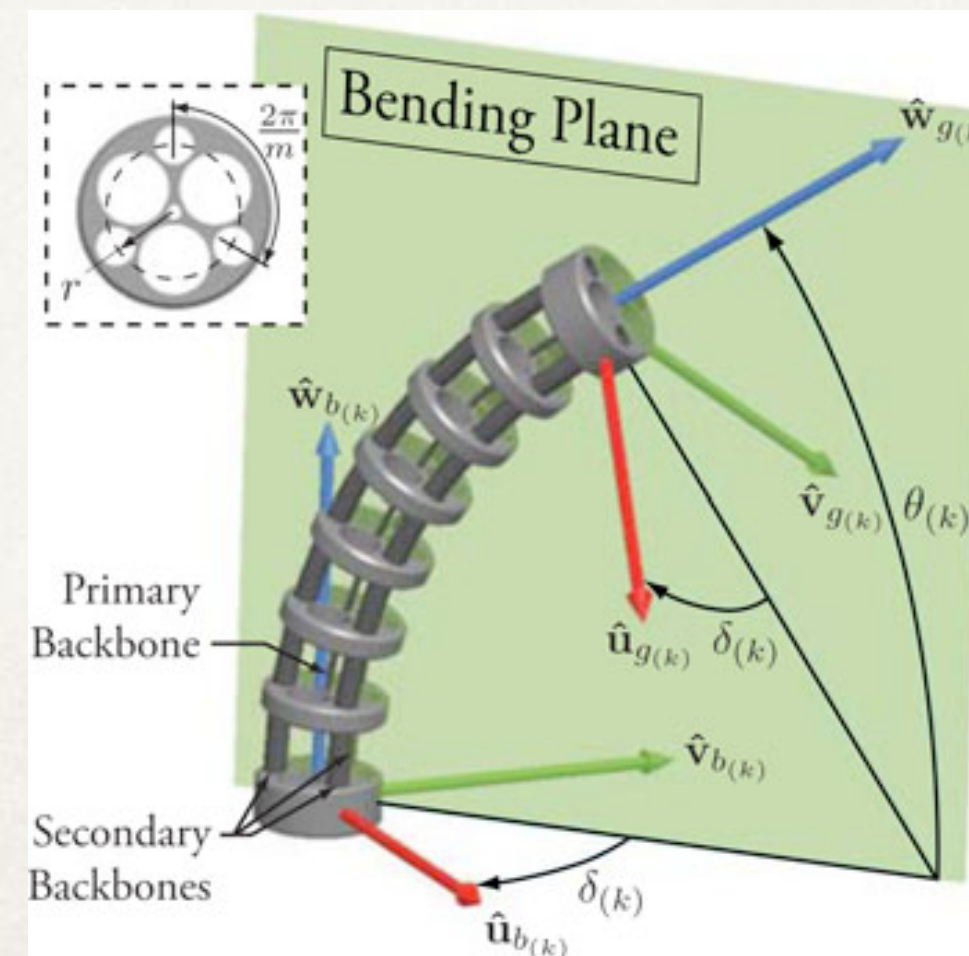
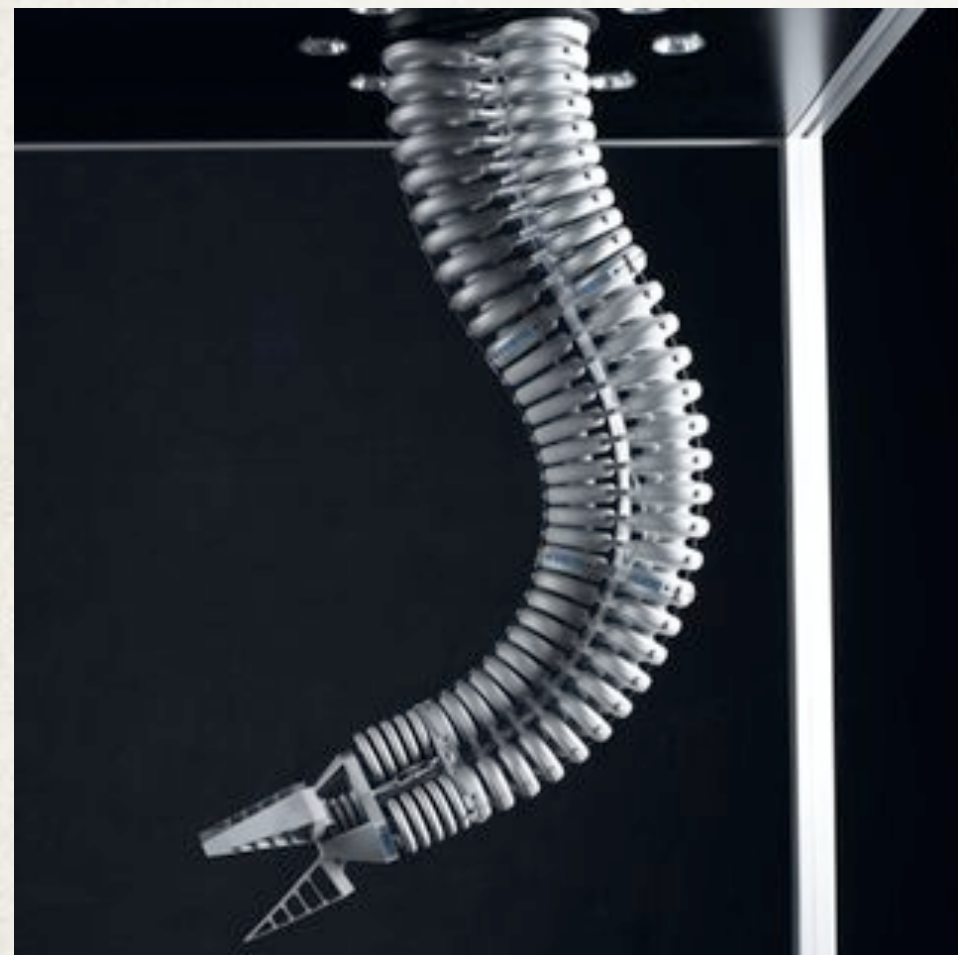
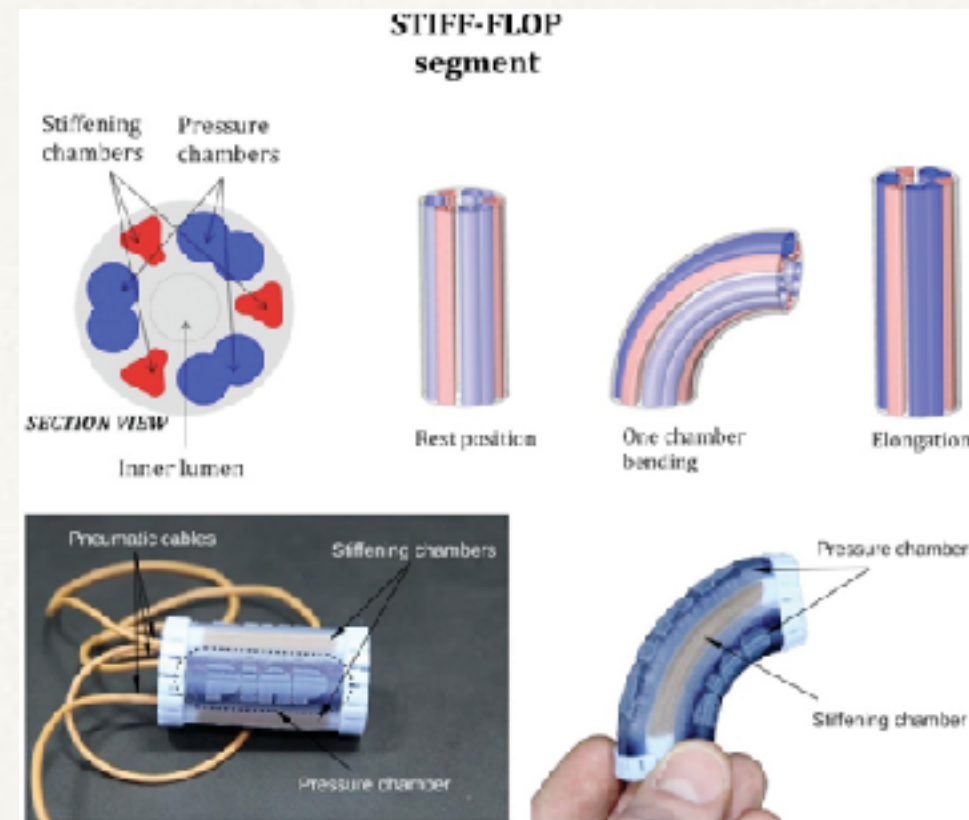
[Dalvand et al., Access 2022]



[Dalvand et al., Access 2022]

current state-of-the-art approach

Abstraction — DACR



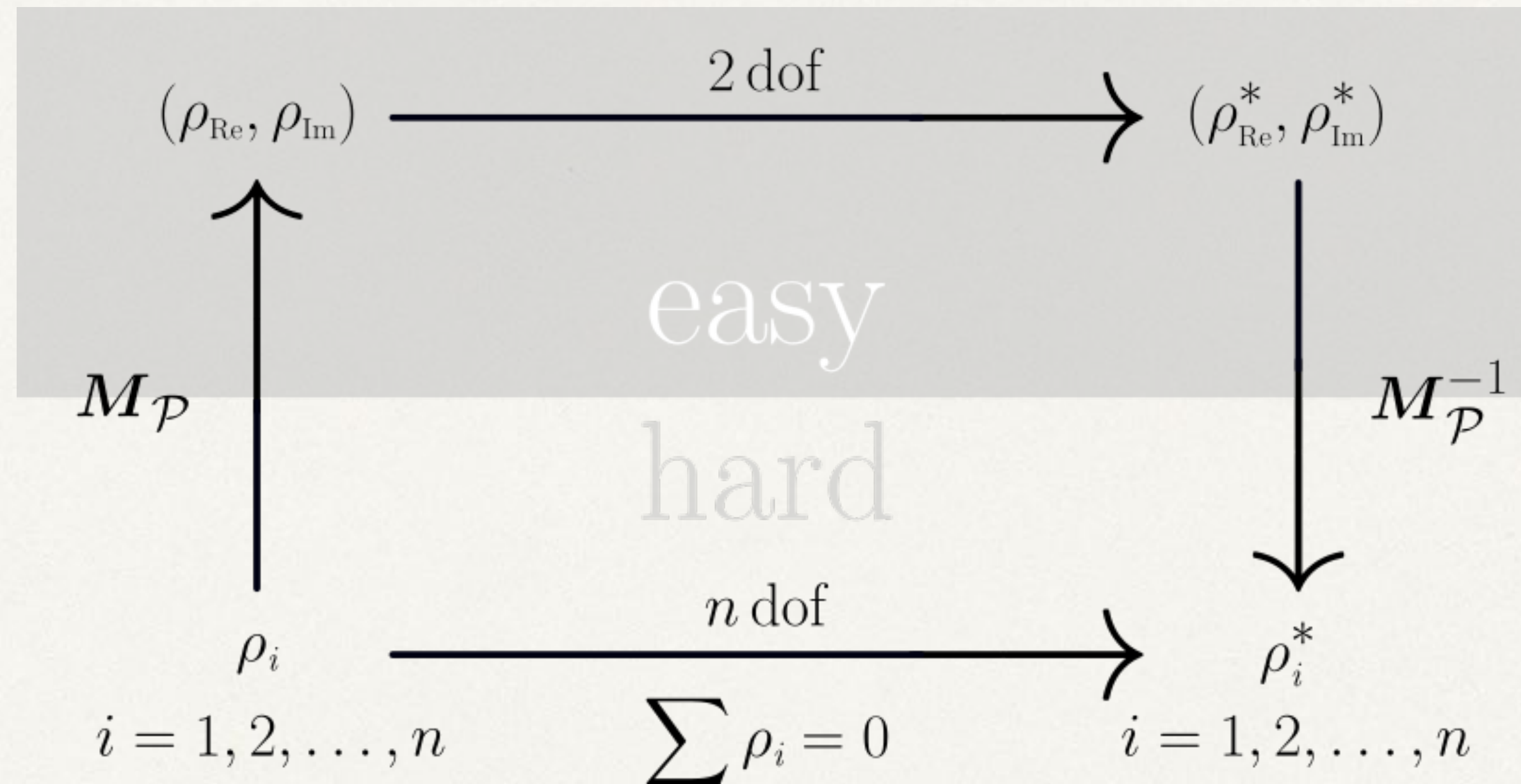
Displacement-Actuated Continuum Robot

[Grassmann et al., arXiv (under review)] "Clarke Transform — A Fundamental Tool for Continuum Robotics"

[Grassmann & Burgner-Kahrs, arXiv (under review)] "Displacement-Actuated Continuum Robot: A Joint Space Abstraction"

Clarke Transform and Clarke Coordinates

Forward
 $\bar{\rho} = M_{\mathcal{P}} \rho \in \mathbb{R}^2$



Backward
 $\rho = M_{\mathcal{P}}^{-1} \bar{\rho} \in \mathbb{R}^n$

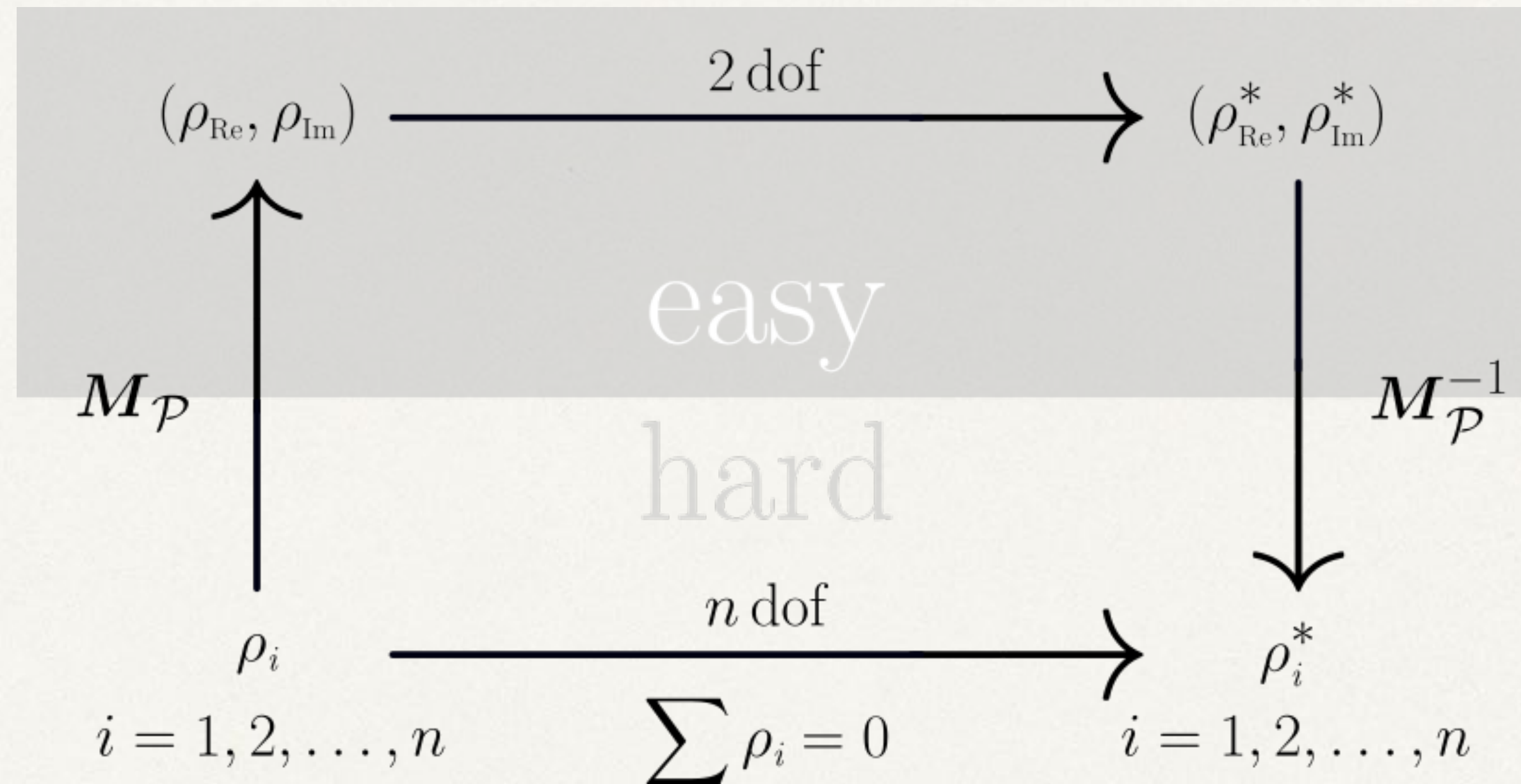


Clarke Transform
(manuscript on arXiv)

Clarke Transform and Clarke Coordinates

Forward

$$\bar{\rho} = M_{\mathcal{P}} \rho \in \mathbb{R}^2$$



Backward

$$\rho = M_{\mathcal{P}}^{-1} \bar{\rho} \in \mathbb{R}^n$$



Clarke Transform
(manuscript on arXiv)

Clarke Transform provide solution that are exact, closed-formed, and interpretable.

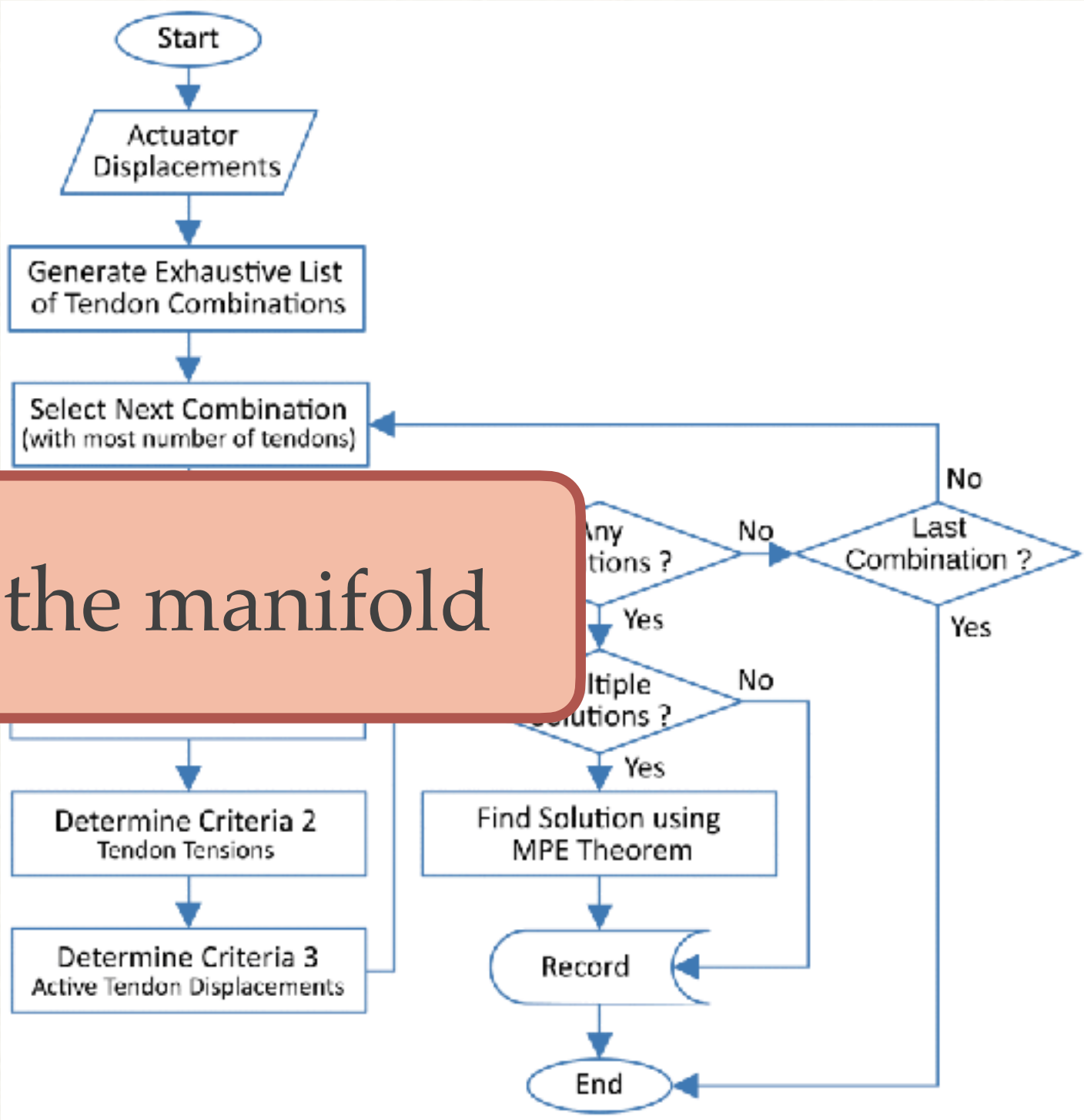
Robot-Dependent Mapping

```
input : Actuator displacements  $[\Delta L_{act}]$ , tendon locations  $[\alpha_{att}], [R_{att}]$  and geometry and mechanical properties of beam and tendons
output: Active/Slack set of tendons and the resulting beam configuration  $([\theta, \phi, L_c])$ 

-21 Algorithm SolveForwardKinematics  $([\Delta L_{act}], [\alpha_{att}], [R_{att}], [properties])$ 
-22
-23  $n \leftarrow size([\alpha_{att}], 1)$ ;
-24 Generate exhaustive list of  $n$ - to 1-tendon combinations;
-25 foreach  $j \in n$  do
-26   foreach  $i \in number\ of\ j\text{-tendon\ combinations}$  do
-27      $\hat{\theta}_0 \leftarrow 0$ ;
-28      $\hat{\phi}_0 \leftarrow 0$ ;
-29      $L_{c0} \leftarrow L_c$ ;
-30     [comb]  $\leftarrow i$ -th  $j$ -tendon combinations;
-31      $[\theta, \phi, L_c] = findBeamConfig([comb], [\hat{\theta}_0, \hat{\phi}_0, L_{c0}], [\Delta L_{act}], [\alpha_{att}], [R_{att}])$ ;
-32     Find resulting tendon displacements  $[\Delta L_t]_{res}$  using Equation (2);
-33      $S_1 \leftarrow Any([\Delta L_t]_{res}$  out of [comb] larger than the corresponding  $[\Delta L_t]$ );
-34     Find resulting [comb] tendon tensions  $[F_t]_{res}$  using Equation (3);
-35      $S_2 \leftarrow Not(Any([F_t]_{res} < 0))$ ;
-36     Find resulting [comb] tendon stretches  $[\delta_t]_{res}$  using Equation (4);
-37     Find resulting [comb] actuator displacements using  $[\Delta L_{at}]_{res} = [\Delta L_t]_{res} + [\delta_t]_{res}$ ;
-38      $S_3 \leftarrow Any([\Delta L_{at}]_{res}$  of [comb] differs from the corresponding  $[\Delta L_{at}]$ );
-39     Compute the beam potential energy using Equation (5);
-40     Record status parameters of  $S_1$ ,  $S_2$  and  $S_3$  as well as the potential energy of this combination;
-41
-42     [sol_index], [sol_comb]  $\leftarrow Find([sol_index], [sol_comb])$ ;
-43     if  $size([sol_index], 1) = 1$  then
-44       solution  $\leftarrow [sol_comb]$ ;
-45       return solution and  $[\theta, \phi, L_c]$ ;
-46     else if  $size([sol_index], 1) > 1$  then
-47       solution  $\leftarrow Find([sol_index], [sol_comb])$ ;
-48       return solution and  $[\theta, \phi, L_c]$ ;
-49     else
-50       Proceed with the next list of  $(j - 1)$ -tendon combinations;
-51
-52 return

-53 Procedure FindBeamConfig  $([comb], [\hat{\theta}_0, \hat{\phi}_0, L_{c0}], [\Delta L_{act}], [\alpha_{att}])$ 
-54
-55 Set  $[\Delta L_{ct}]$  based on [comb] and  $[\Delta L_{act}]$ ;
-56 Set  $[\alpha_t]$  based on [comb] and  $[\alpha_{att}]$ ;
-57 Set  $[R_t]$  based on [comb] and  $[R_{att}]$ ;
-58 Derive tendon displacements  $[\Delta L_t]$  using Equation (2);
-59 Derive tendon tensions  $[F_t]$  using Equation (3);
-60 Derive tendon stretches  $[\delta_t]$  using Equation (4);
-61 Define equations  $[Eq_t] = [\Delta L_{at}] - [\Delta L_t] - [\delta_t]$ ;
-62 Find beam configuration  $[\theta, \phi, L_c]$  by numerically solving the set of Equations  $[Eq_t] = 0$  based on the initial condition  $[\hat{\theta}_0, \hat{\phi}_0, L_{c0}]$ ;
-63 return  $[\theta, \phi, L_c]$ ;
```

without utilizing the manifold



[Dalvand et al., Access 2022]

[Dalvand et al., Access 2022]

Robot-Dependent Mapping

input : Actuator displacements $([\Delta L_{act}])$, tendon locations $([\alpha_{att}], [R_{att}])$ and geometry and mechanical properties of beam and tendons
output: Active/Slack set of tendons and the resulting beam configuration $([\theta, \phi, L_c])$

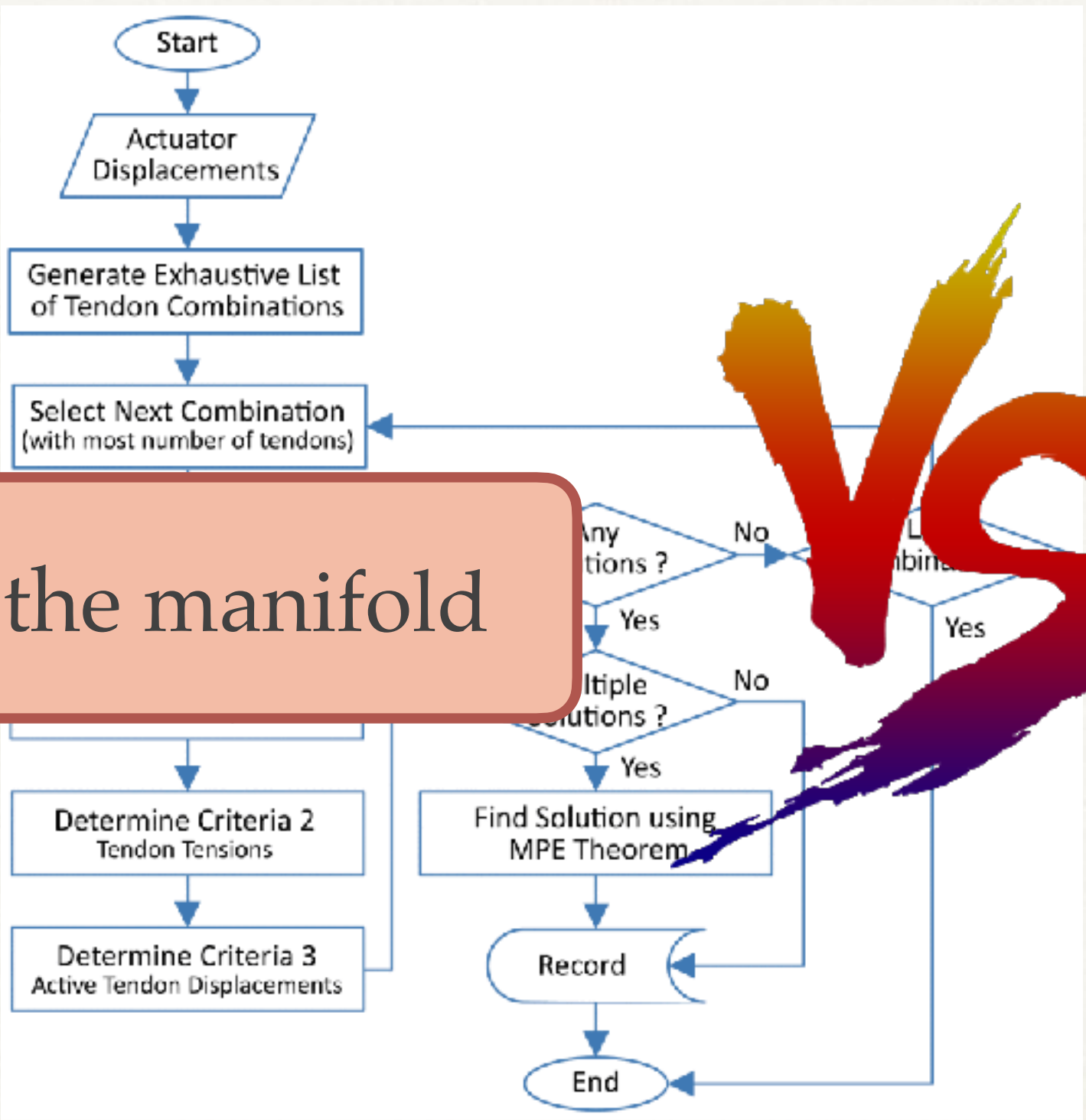
Algorithm SolveForwardKinematics $([\Delta L_{act}], [\alpha_{att}], [R_{att}], [properties])$

```
21 Algorithm SolveForwardKinematics  $([\Delta L_{act}], [\alpha_{att}], [R_{att}], [properties])$ 
22
23  $n \leftarrow \text{size}([\alpha_{att}], 1)$ ;
24 Generate exhaustive list of  $n$ - to 1-tendon combinations;
25 foreach  $j \in n$  do
26   foreach  $i \in \text{number of } j\text{-tendon combinations}$  do
27      $\hat{\theta}_0 \leftarrow 0$ ;
28      $\hat{\phi}_0 \leftarrow 0$ ;
29      $L_{c0} \leftarrow L_c$ ;
30      $[comb] \leftarrow i\text{-th } j\text{-tendon combinations}$ ;
31      $[\theta, \phi, L_c] = \text{findBeamConfig}([comb], [\hat{\theta}_0, \hat{\phi}_0, L_{c0}], [\Delta L_{act}], [\alpha_{att}], [R_{att}])$ ;
32     Find resulting tendon displacements  $[\Delta L_i]_{res}$  using Equation (2);
33      $S_1 \leftarrow \text{Any}([\Delta L_i]_{res} \text{ out of } [comb] \text{ larger than the corresponding } [\Delta L_i])$ ;
34     Find resulting  $[comb]$  tendon tensions  $[F_i]_{res}$  using Equation (3);
35      $S_2 \leftarrow \text{Not}(\text{Any}([F_i]_{res} < 0))$ ;
36     Find resulting  $[comb]$  tendon stretches  $[\delta_i]_{res}$  using Equation (4);
37     Find resulting  $[comb]$  actuator displacements using  $[\Delta L_{act}]_{res} = [\Delta L_i]_{res} + [\delta_i]_{res}$ ;
38      $S_3 \leftarrow \text{Any}([\Delta L_{act}]_{res} \text{ of } [comb] \text{ differs from the corresponding } [\Delta L_{act}])$ ;
39     Compute the beam potential energy using Equation (5);
40     Record status parameters of  $S_1$ ,  $S_2$ , and  $S_3$  as well as the potential energy of this combination;
41
42    $[sol_{index}], [sol_{comb}] \leftarrow \text{Find}([comb], [S_1], [S_2], [S_3])$ ;
43   if  $\text{size}([sol_{index}], 1) = 1$  then
44      $solution \leftarrow [sol_{comb}]$ ;
45     return  $solution$  and  $[\theta, \phi, L_c]$ ;
46   else if  $\text{size}([sol_{index}], 1) > 1$  then
47      $solution \leftarrow \text{Find}([sol_{comb}], [S_1], [S_2], [S_3])$ ;
48     return  $solution$  and  $[\theta, \phi, L_c]$ ;
49   else
50     Proceed with the next list of  $(j-1)$ -tendon combinations;
51
52 return
```

Procedure FindBeamConfig $([comb], [\hat{\theta}_0, \hat{\phi}_0, L_{c0}], [\Delta L_{act}], [\alpha_{att}])$

```
231 Procedure FindBeamConfig  $([comb], [\hat{\theta}_0, \hat{\phi}_0, L_{c0}], [\Delta L_{act}], [\alpha_{att}])$ 
232
233 Set  $[\Delta L_{c0}]$  based on  $[comb]$  and  $[\Delta L_{act}]$ ;
234 Set  $[\alpha_i]$  based on  $[comb]$  and  $[\alpha_{att}]$ ;
235 Set  $[R_i]$  based on  $[comb]$  and  $[R_{att}]$ ;
236 Derive tendon displacements  $[\Delta L_i]$  using Equation (2);
237 Derive tendon tensions  $[F_i]$  using Equation (3);
238 Derive tendon stretches  $[\delta_i]$  using Equation (4);
239 Define equations  $[Eq_i] = [\Delta L_{act}] - [\Delta L_i] - [\delta_i]$ ;
240 Find beam configuration  $[\theta, \phi, L_c]$  by numerically solving the set of Equations  $[Eq_i] = 0$  based on the initial condition  $[\hat{\theta}_0, \hat{\phi}_0, L_{c0}]$ ;
241 return  $[\theta, \phi, L_c]$ ;
```

without utilizing the manifold



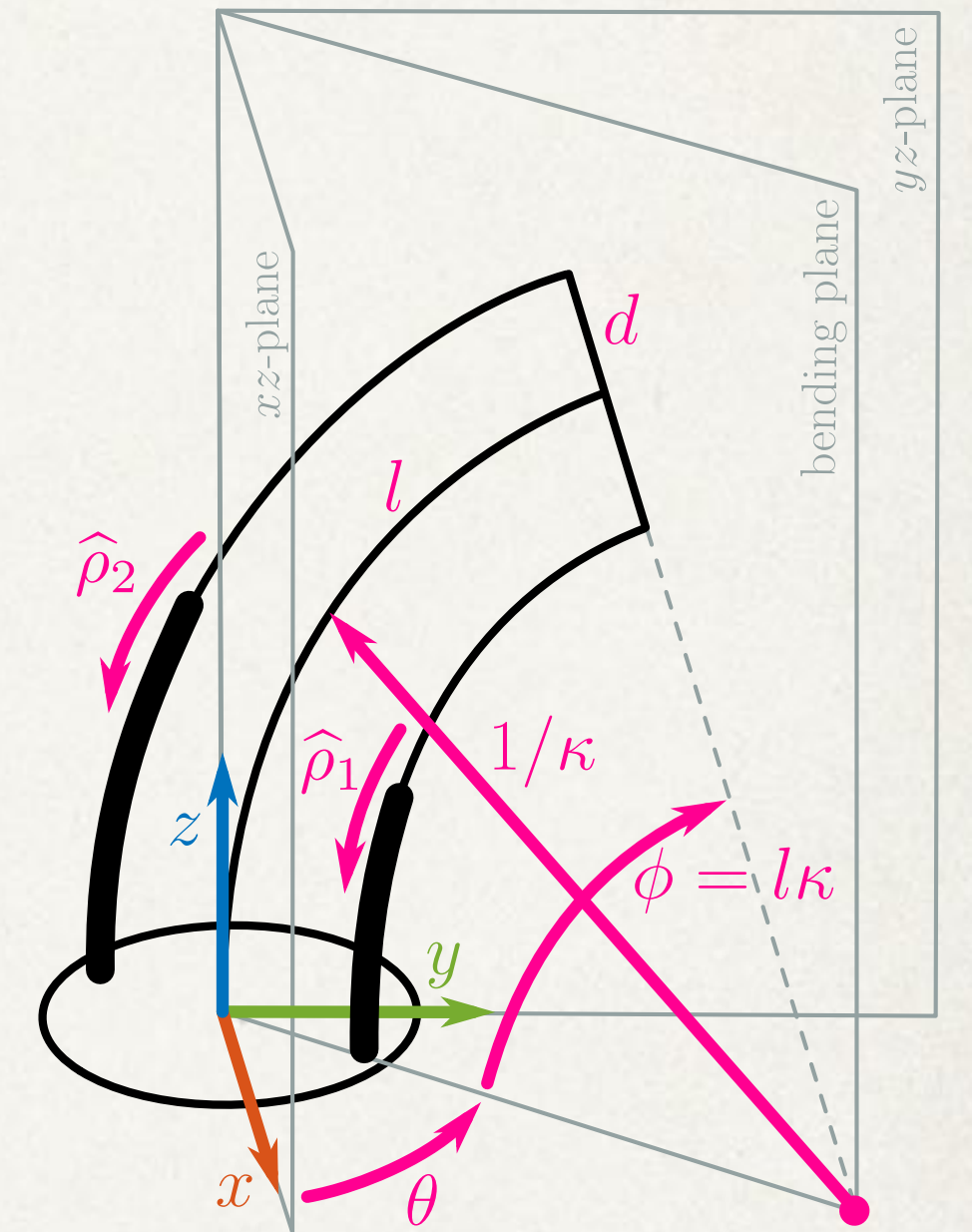
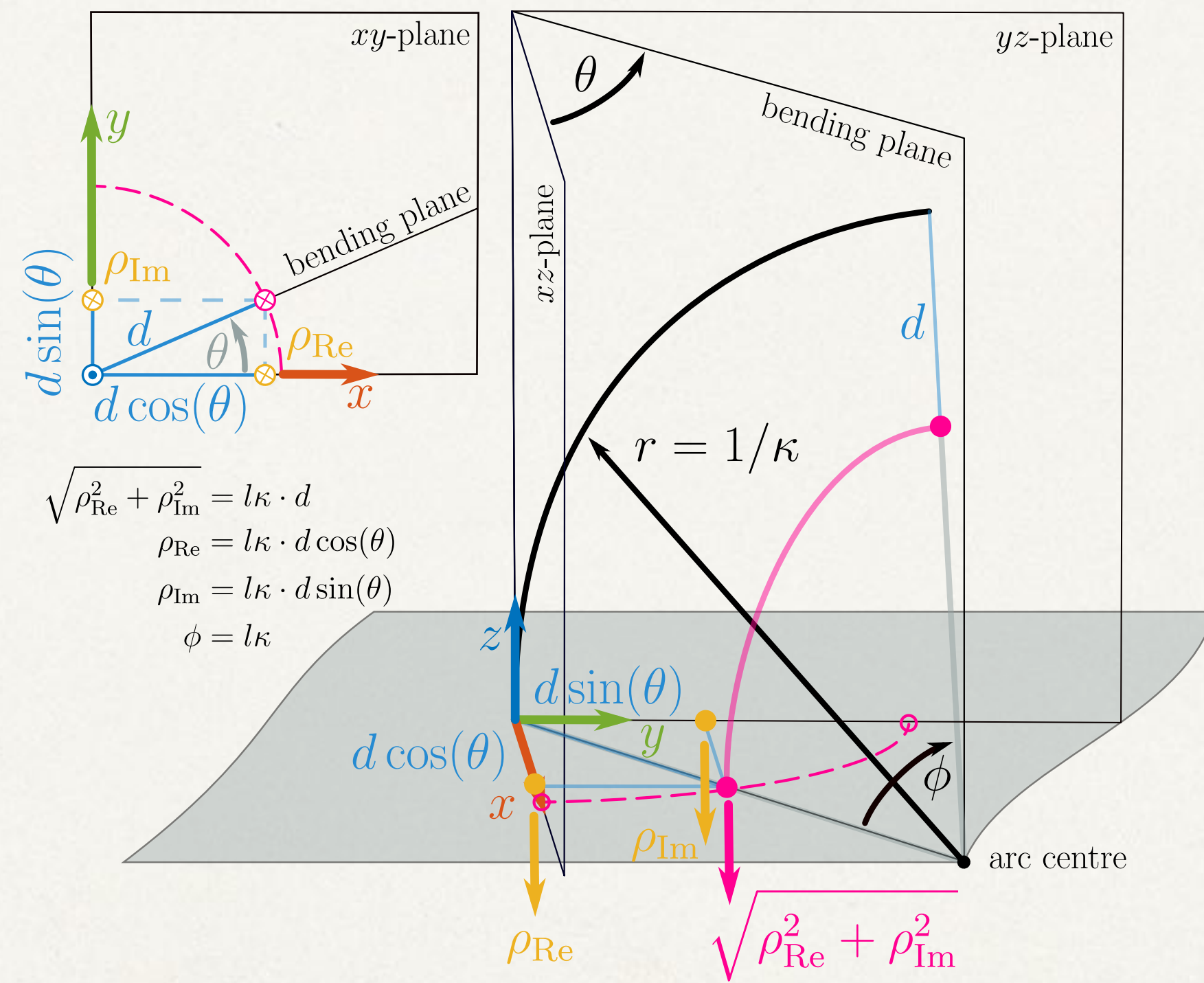
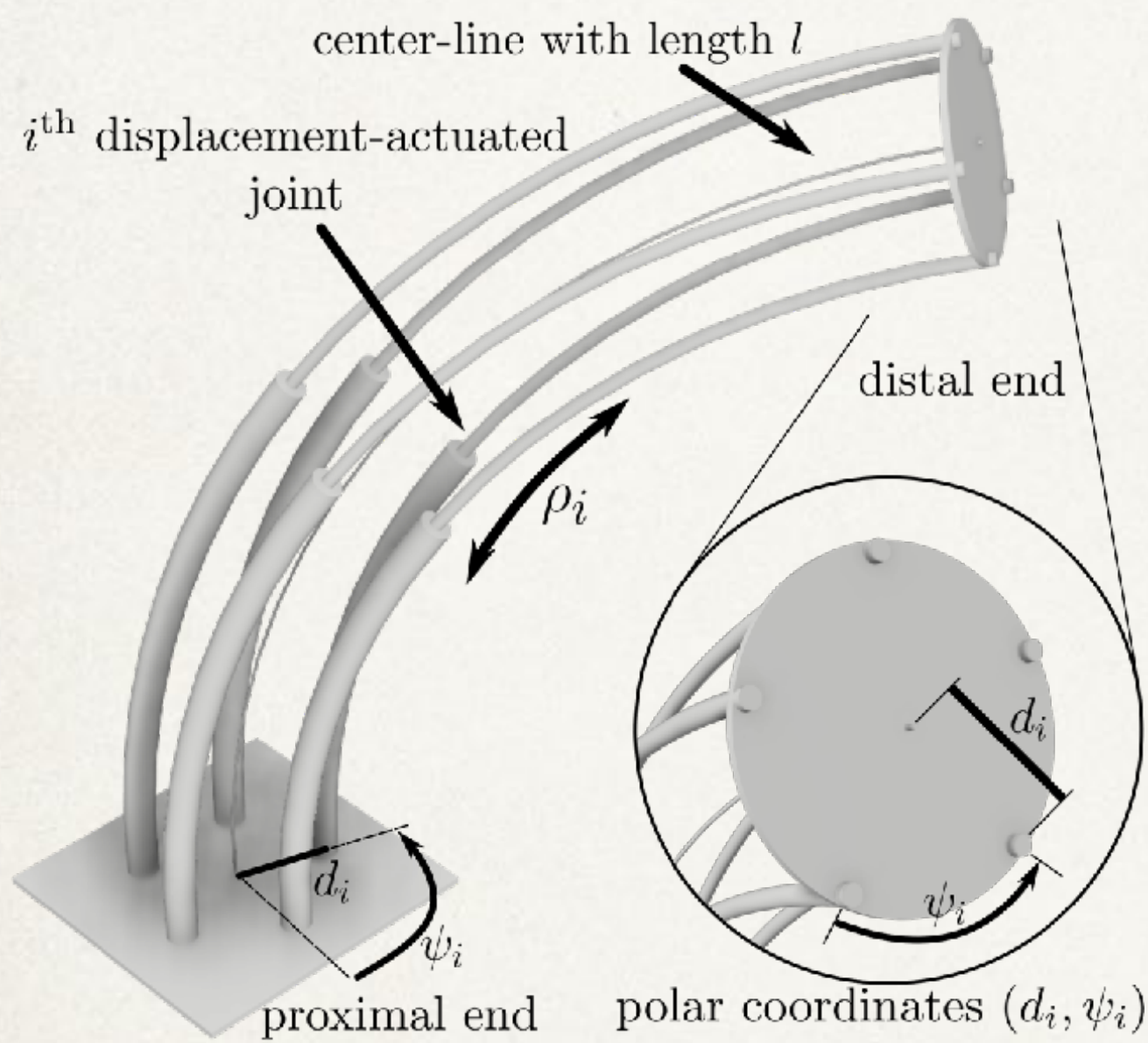
[Dalvand et al., Access 2022]

[Dalvand et al., Access 2022]

$$\begin{bmatrix} \kappa \cos(\theta) \\ \kappa \sin(\theta) \end{bmatrix} = \underbrace{1/l}_{\text{removes } l} \underbrace{\widehat{M}_{\mathcal{P}}}_{\text{removes } \psi_i} \underbrace{\text{diag}(1/d_i)}_{\text{removes } d_i} \rho$$

utilizing the manifold

Interpretable



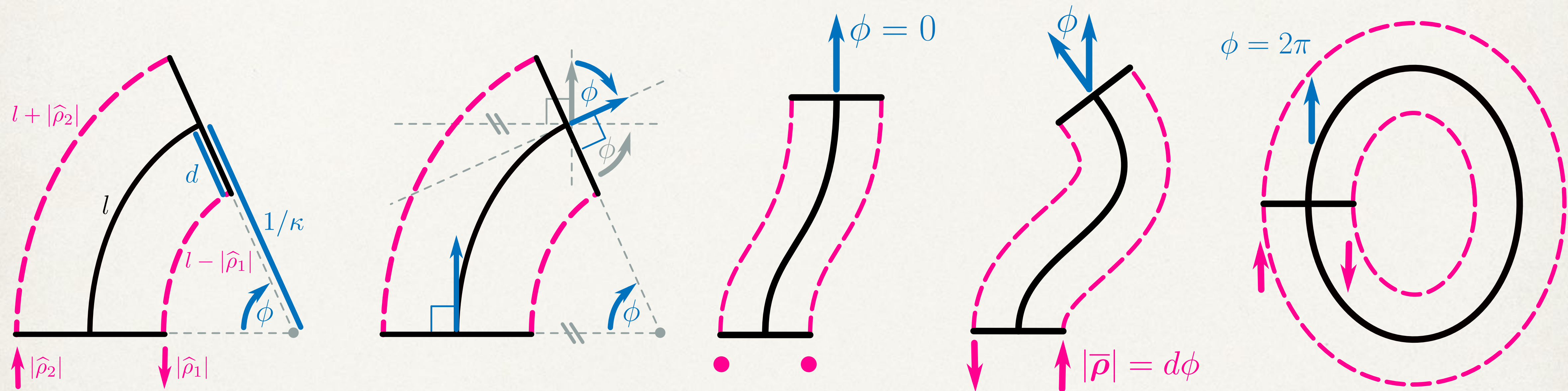
[Grassmann et al., arXiv (under review)] “Clarke Transform — A Fundamental Tool for Continuum Robotics”

[Grassmann & Burgner-Kahrs, arXiv (under review)] “Using Clarke Transform to Create a Framework on the Manifold”

[Grassmann & Burgner-Kahrs, arXiv (under review)] “Clarke Coordinates Are Generalized Improved State Parametrization for Continuum Robots”

[Grassmann & Burgner-Kahrs, arXiv (under review)] “Displacement-Actuated Continuum Robots: A Joint Space Abstraction”

Applicable Beyond Constant-Curvature



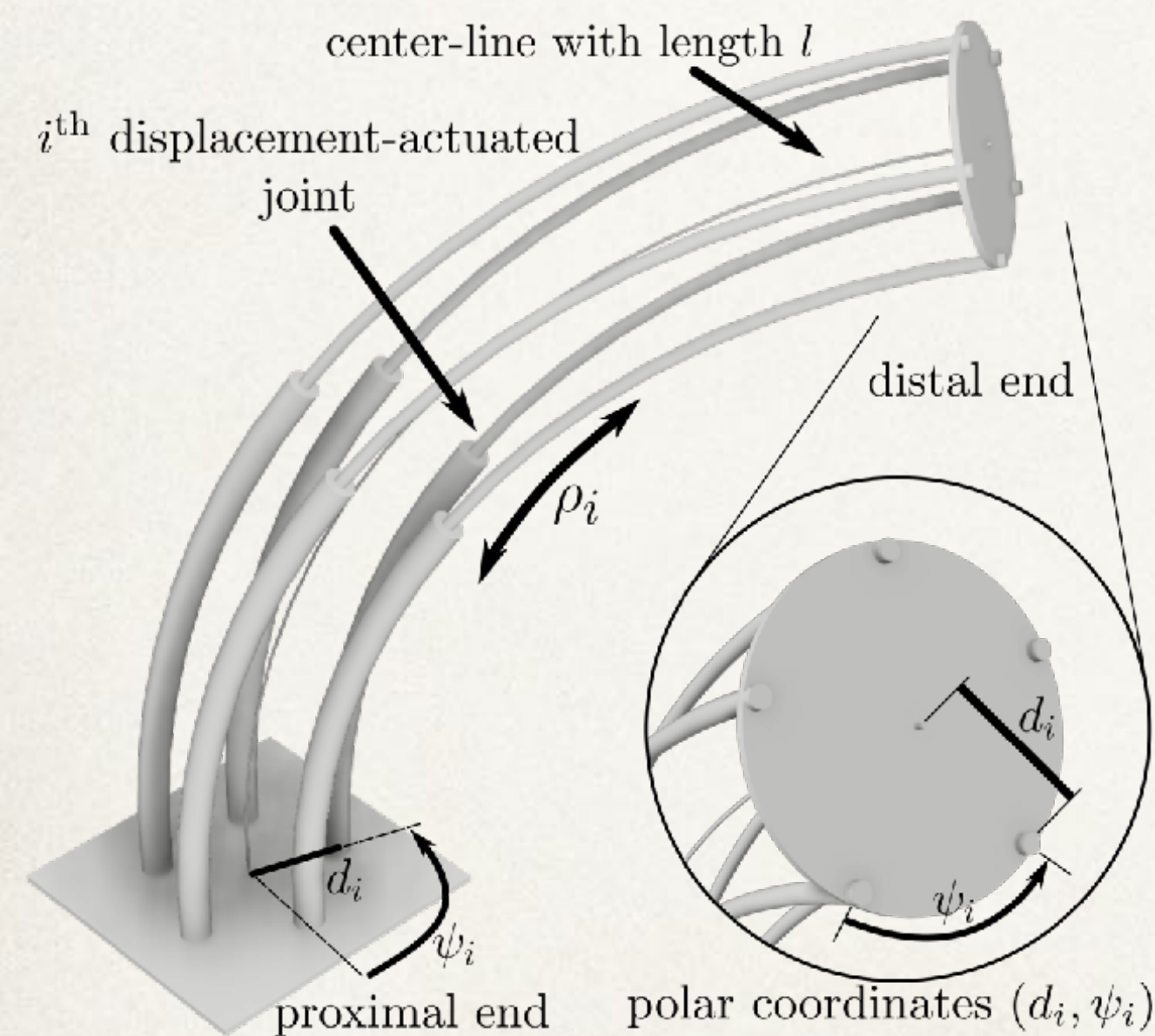
$$\begin{bmatrix} d\phi \cos(\theta) \\ d\phi \sin(\theta) \end{bmatrix} = \bar{\rho} = M_{\mathcal{P}} \rho$$

Improved State Parameterizations

Reference	n	Parameterization w.r.t. joint values	Parameterization w.r.t. Clarke Coordinates
Della Santina <i>et al.</i>	4	$\Delta_x = \frac{l_3 - l_1}{2}$ $\Delta_y = \frac{l_4 - l_2}{2}$	$\Delta_x = \rho_{\text{Re}}$ $\Delta_y = \rho_{\text{Im}}$
Allen <i>et al.</i>	3	$u = \frac{l_2 - l_3}{\sqrt{3}d}$ $v = \frac{(l_1 + l_2 + l_3)/3 - l_1}{d}$	$v = (1/d) \rho_{\text{Re}}$ $u = (1/d) \rho_{\text{Im}}$
Allen <i>et al.</i>	4	$u = \frac{l_2 - l_4}{d}$ $v = \frac{l_3 - l_1}{d}$	$v = (2/d) \rho_{\text{Re}}$ $u = (2/d) \rho_{\text{Im}}$
Dian <i>et al.</i>	3	$\Delta x = \frac{l_2 + l_3 - 2l_1}{3}$ $\Delta y = \frac{l_3 - l_2}{\sqrt{3}}$	$\Delta x = \rho_{\text{Re}}$ $\Delta y = \rho_{\text{Im}}$
Grassmann <i>et al.</i>	n	$[\rho_{\text{Re}}, \rho_{\text{Im}}]^\top = \mathbf{M}_{\mathcal{P}} \boldsymbol{\rho}$	ρ_{Re} ρ_{Im}

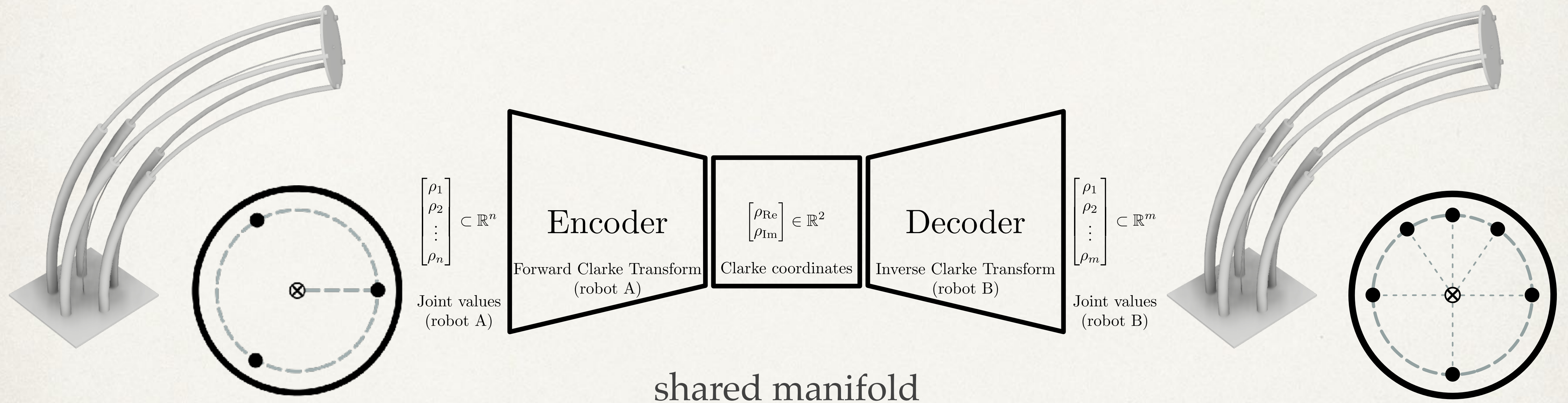
[Della Santina et al., RA-L 2020] “On an Improved State Parametrization for Soft Robots with Piecewise Constant Curvature and Its Use in Model-Based Control”
 [Allen et al., RoboSoft 2020] “Closed-Form Non-Singular Constant-Curvature Continuum Manipulator Kinematics”
 [Dian et al., Access 2022] “A Novel Disturbance-Rejection Control Framework for Cable-Driven Continuum Robots With Improved State Parameterization”
 [Grassmann & Burgner-Kahrs, arXiv (under review)] “Clarke Coordinates Are Generalized Improved State Parametrization for Continuum Robots”

Arbitrary Joint Locations



joint location					
	Allen <i>et al.</i>	Della Santina <i>et al.</i>	Grassmann <i>et al.</i>	Grassmann <i>et al.</i>	Ours
arbitrary joint number n					
arbitrary distance d_i					
arbitrary angle ψ_i					

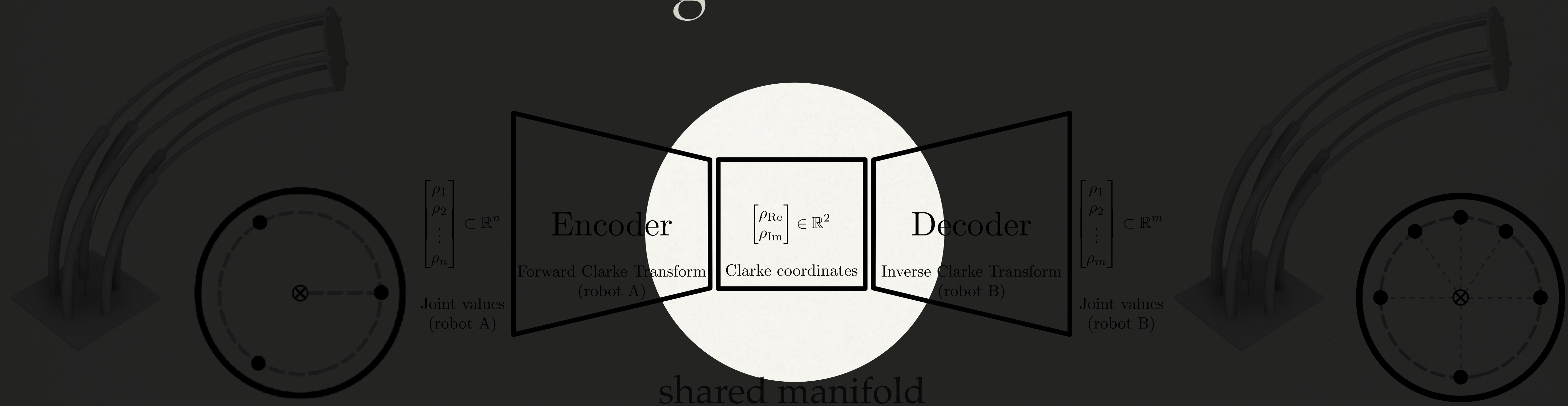
Re-using Previous Approaches



$$\boldsymbol{\rho}_{(\text{robot B})} = \overbrace{l_{(\text{robot B})} \text{diag} \left(d_{i,(\text{robot B})} \right) \mathbf{M}_{\mathcal{P}}^{-1}(\text{robot B})}^{\text{adds design parameters of robot B}} \cdot \underbrace{\frac{1}{l_{(\text{robot A})}} \mathbf{M}_{\mathcal{P}}(\text{robot A}) \text{diag} \left(\frac{1}{d_{i,(\text{robot A})}} \right)}_{\text{removes design parameters of robot A}} \boldsymbol{\rho}_{(\text{robot A})}$$

Re-using Previous Approaches

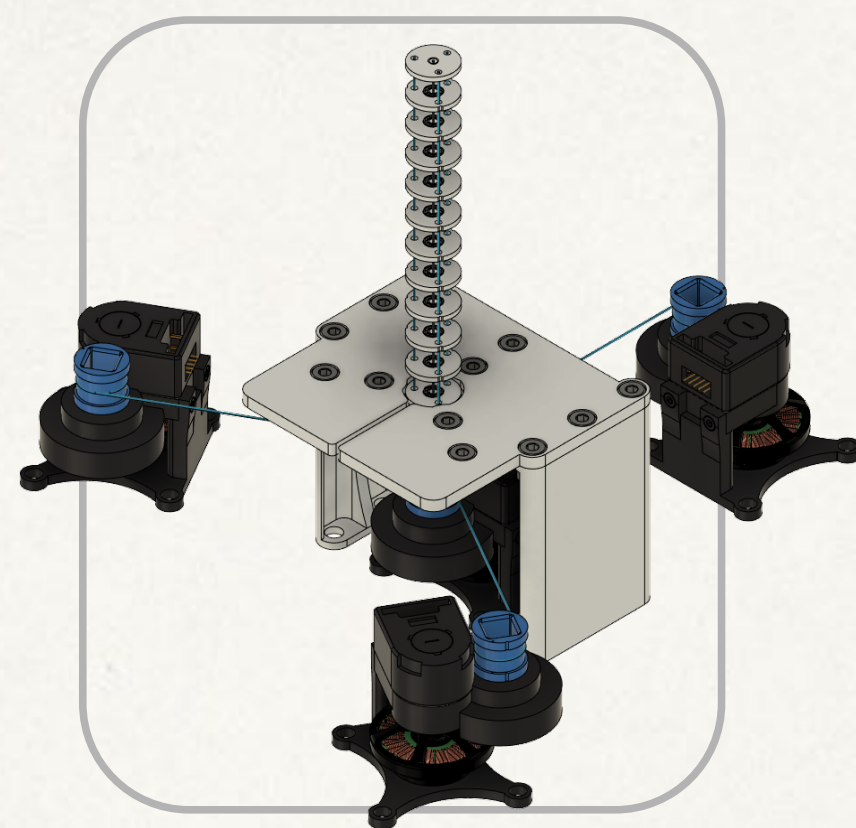
Lingua Franca



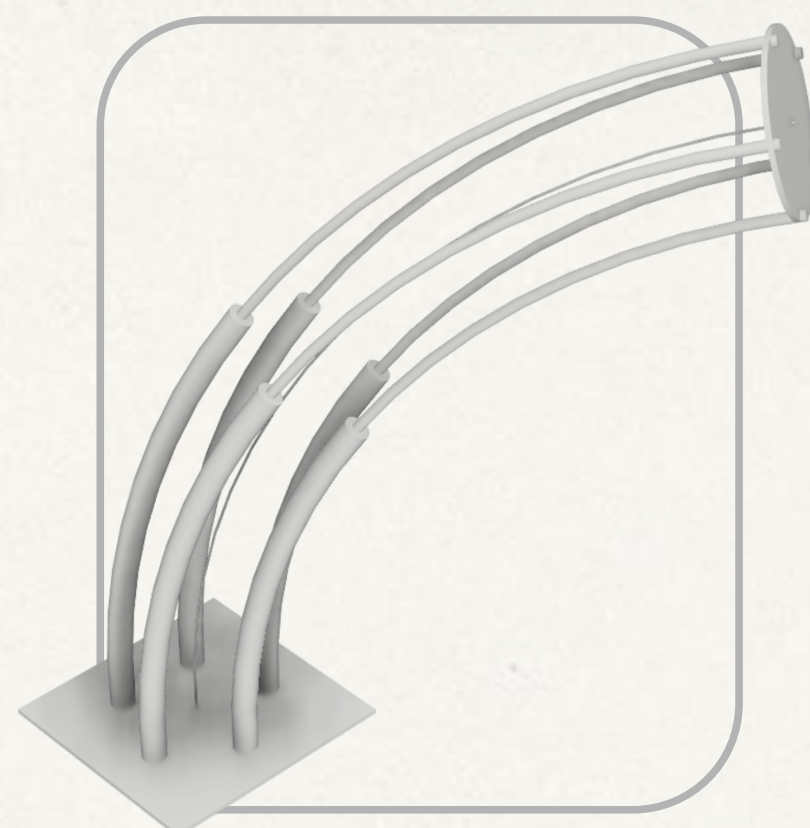
adds design parameters of robot B

$$\rho_{(\text{robot B})} = \underbrace{l_{(\text{robot A})} \underbrace{M_{\mathcal{P}}^{-1}(\text{robot A})}_{\text{removes design parameters of robot A}} \text{diag} \left(\frac{1}{d_{i,(\text{robot A})}} \right)}_{\text{removes design parameters of robot A}} \rho_{(\text{robot A})}$$

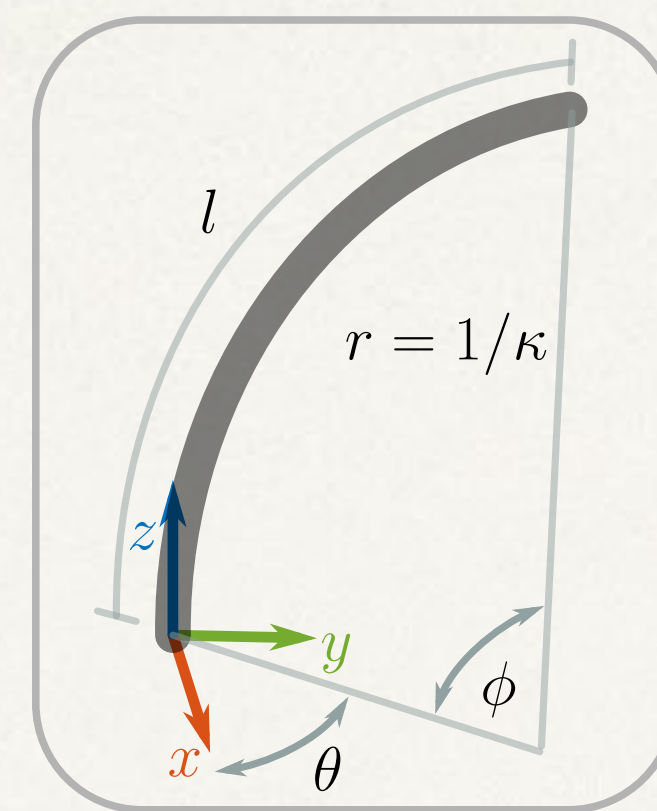
removes design parameters of robot A



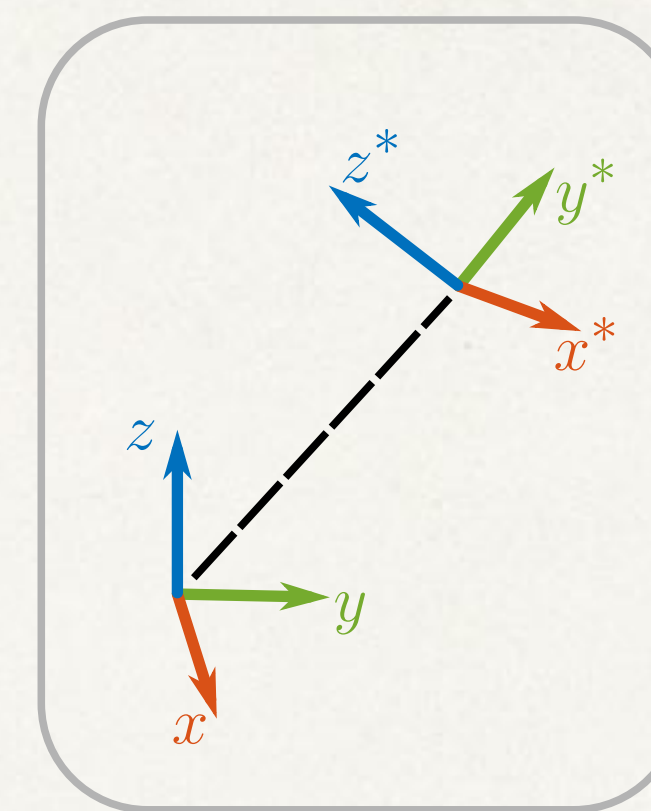
Physical Hardware



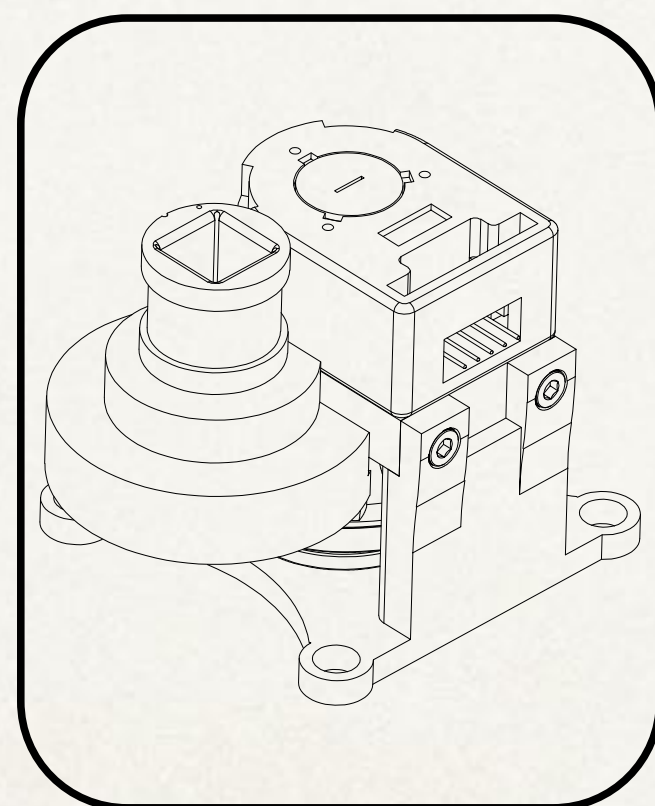
Displacement-Actuated
Continuum Robot



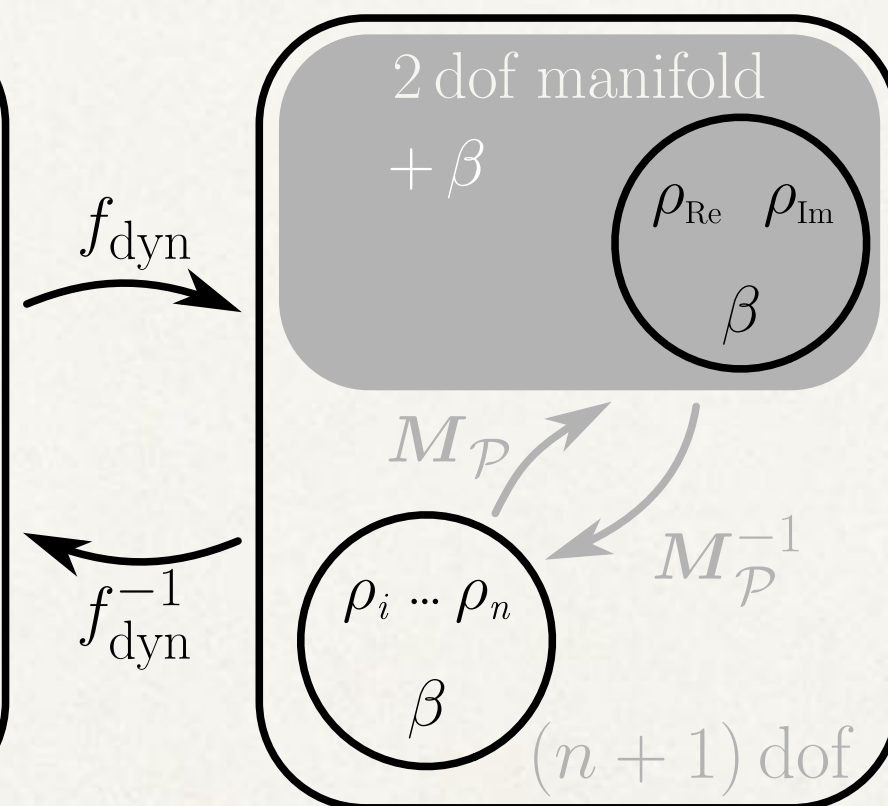
Constant Curvature
Assumption



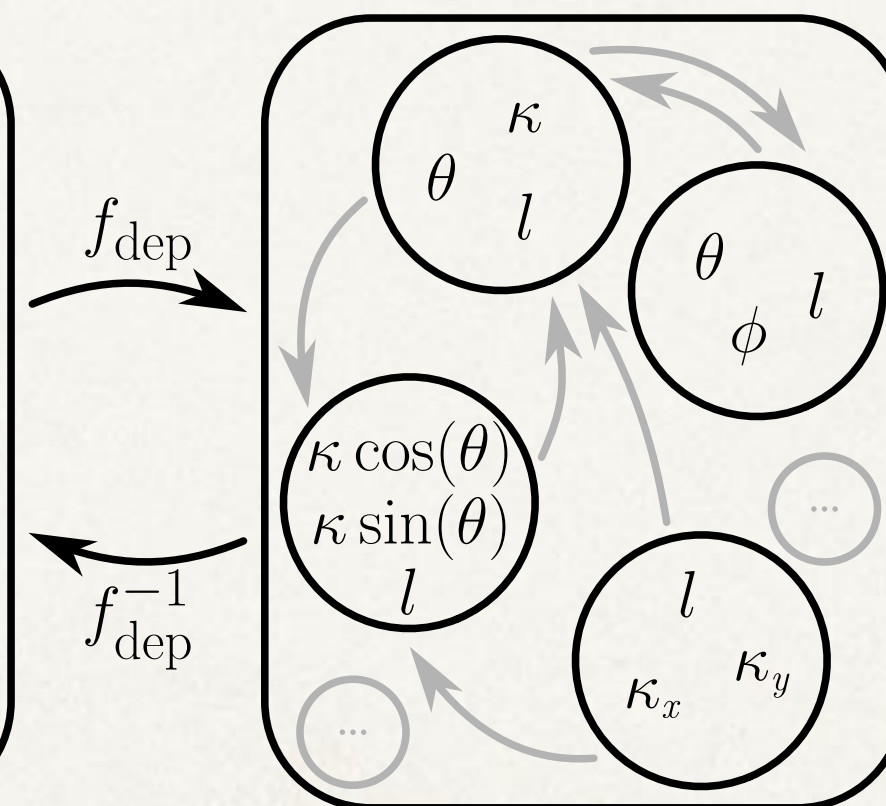
Rigid Transformation



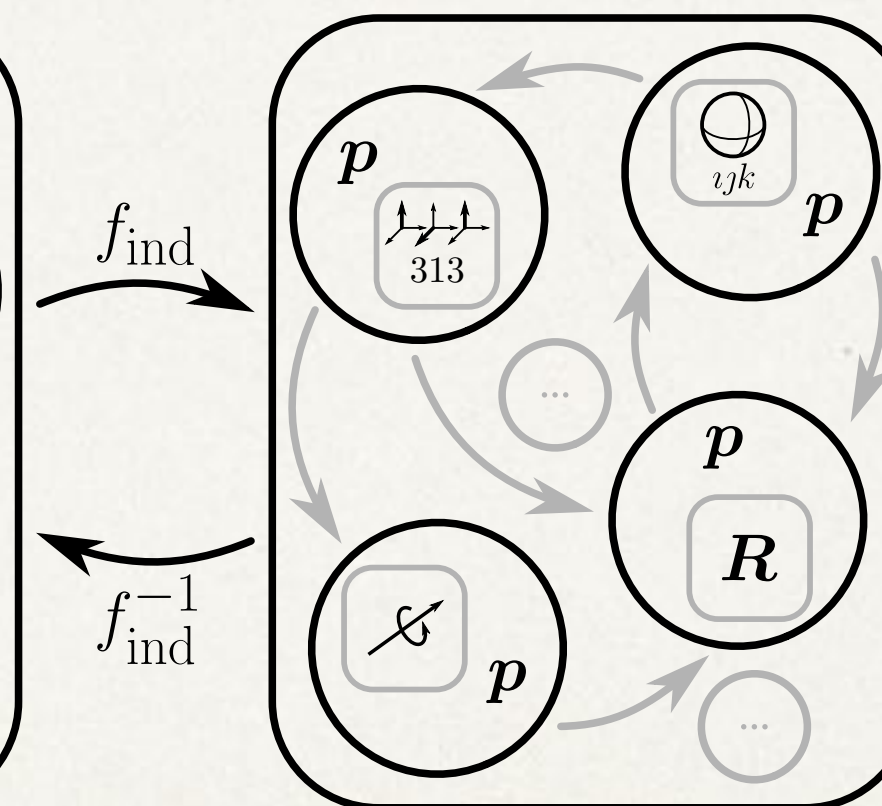
Actuation Space



Joint Space



Arc Space



Task Space

f_{dyn}

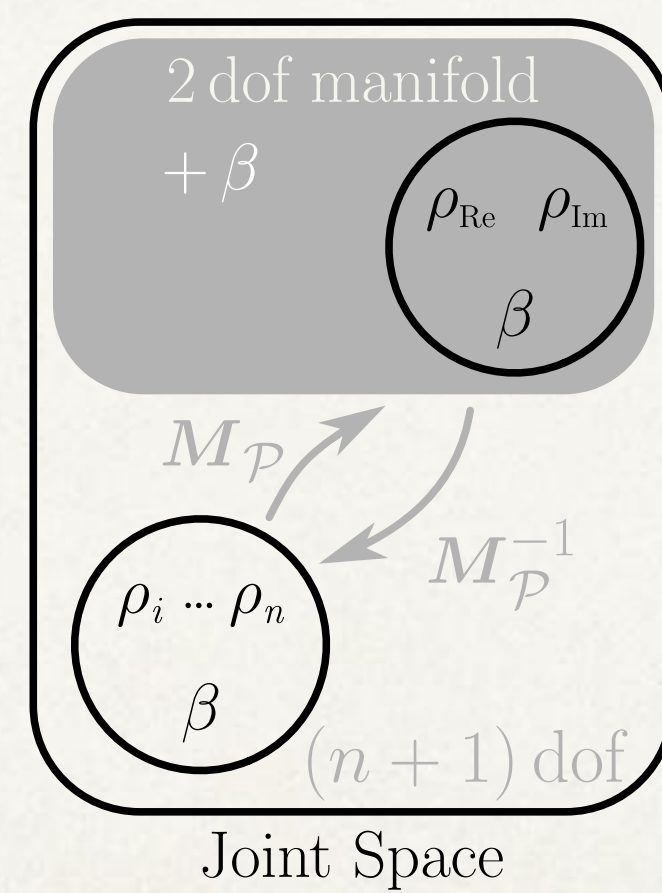
f_{dyn}^{-1}

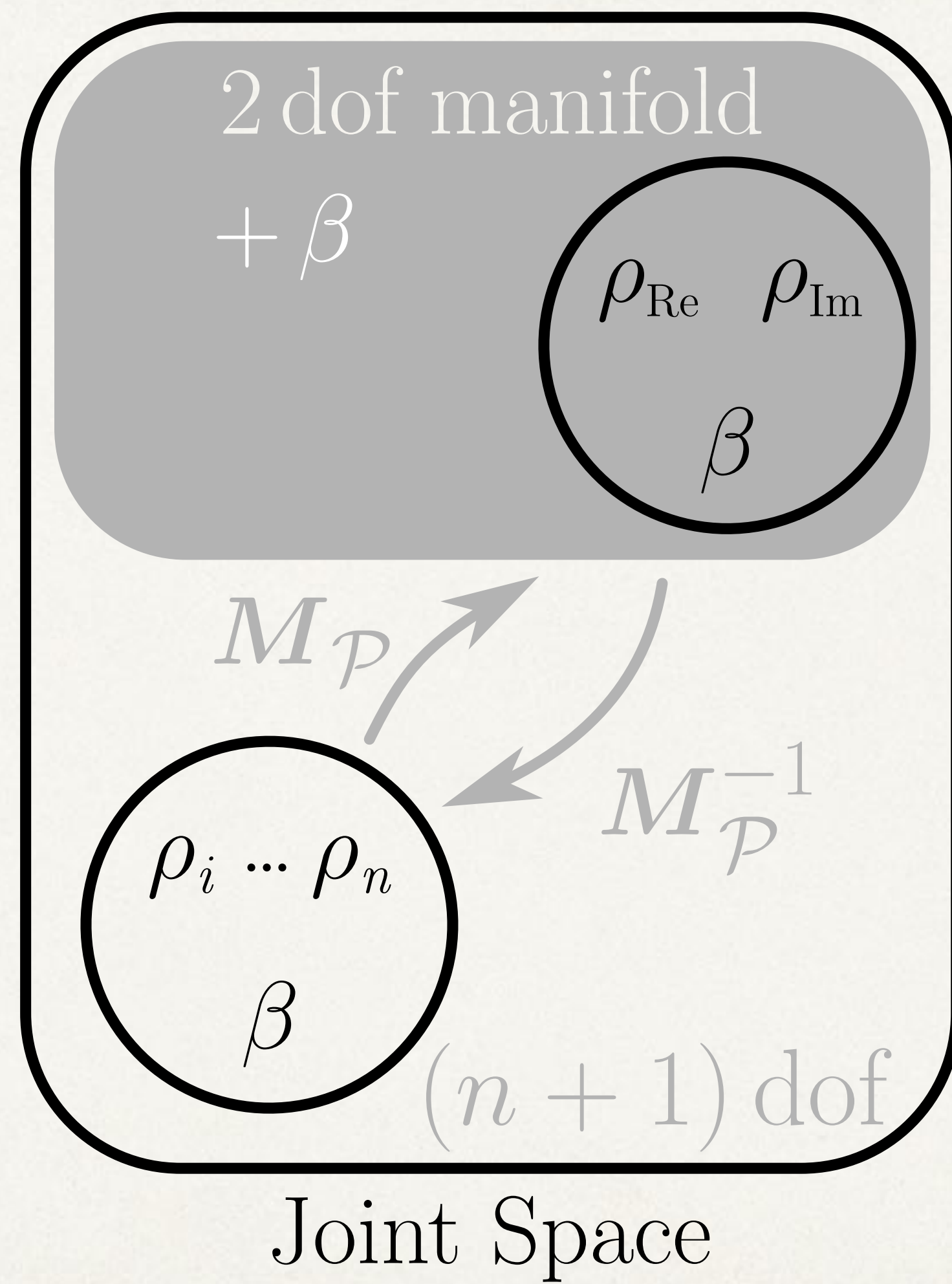
f_{dep}

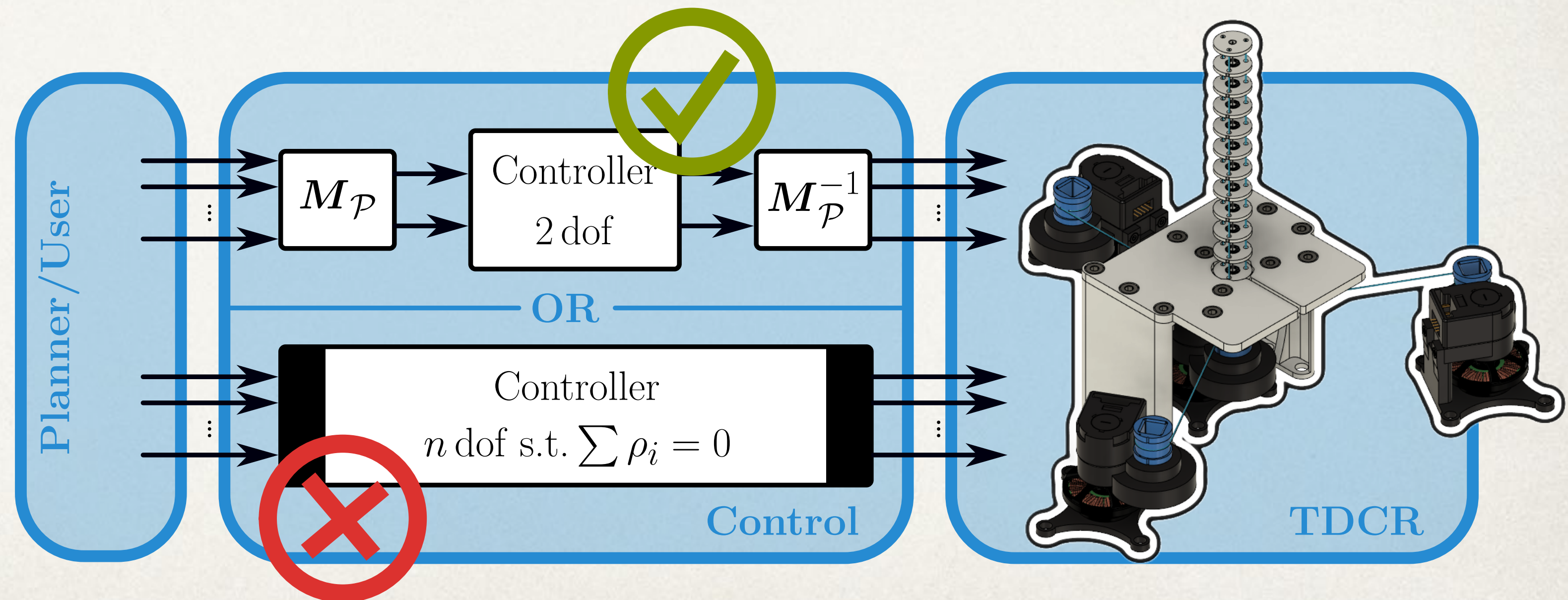
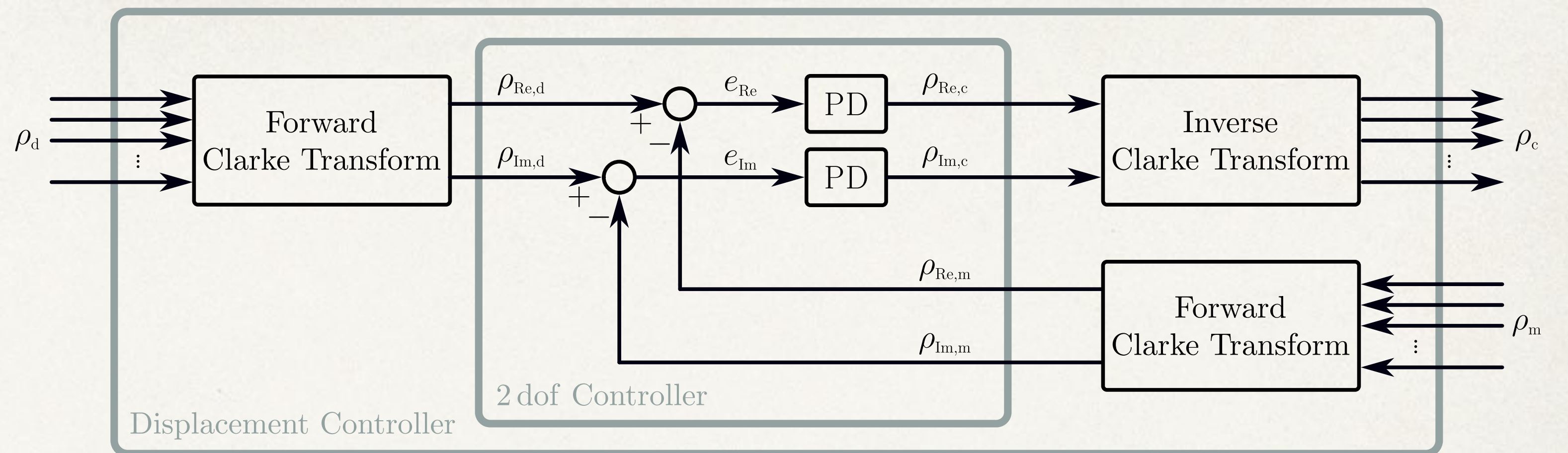
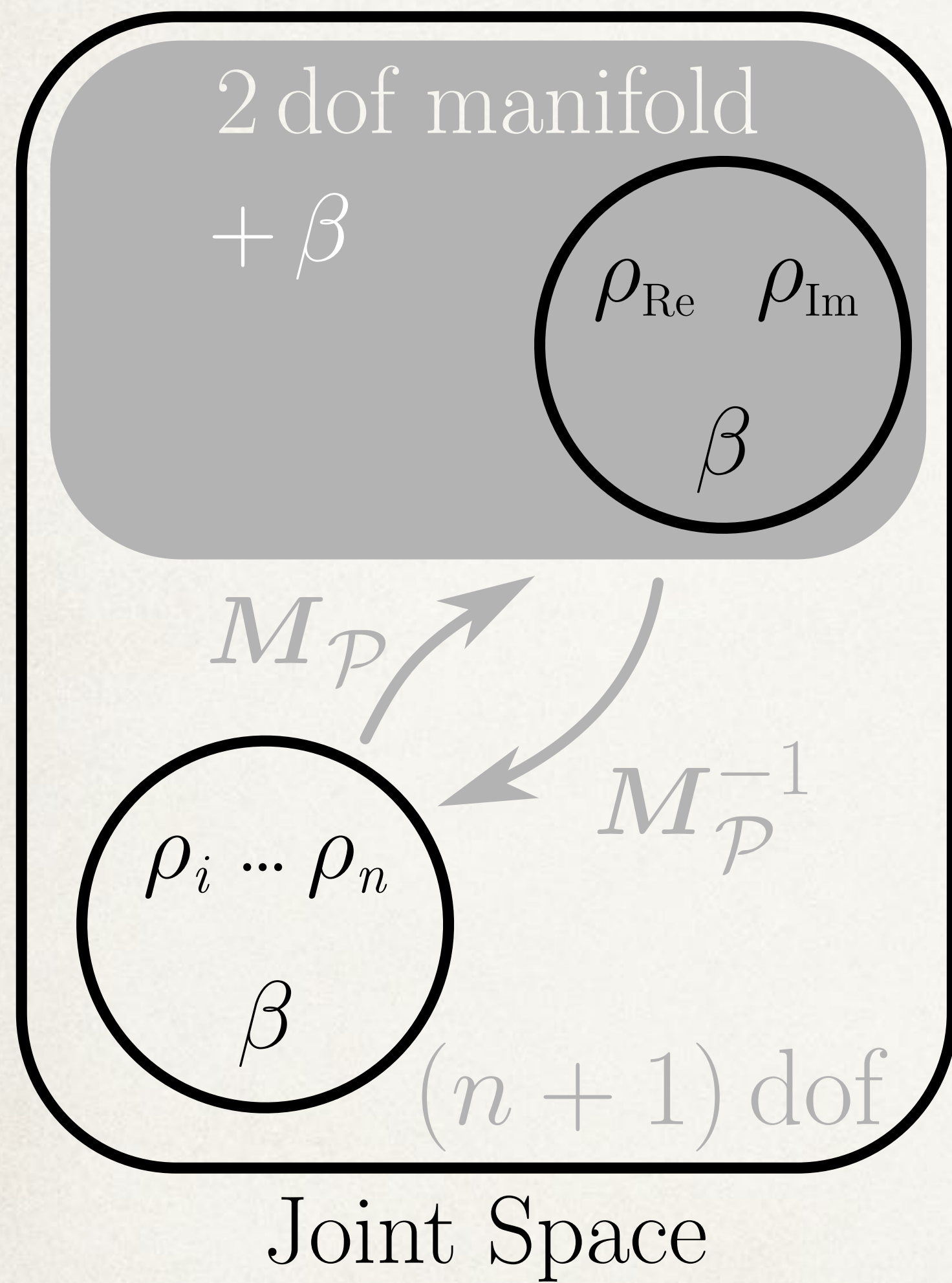
f_{dep}^{-1}

f_{ind}

f_{ind}^{-1}



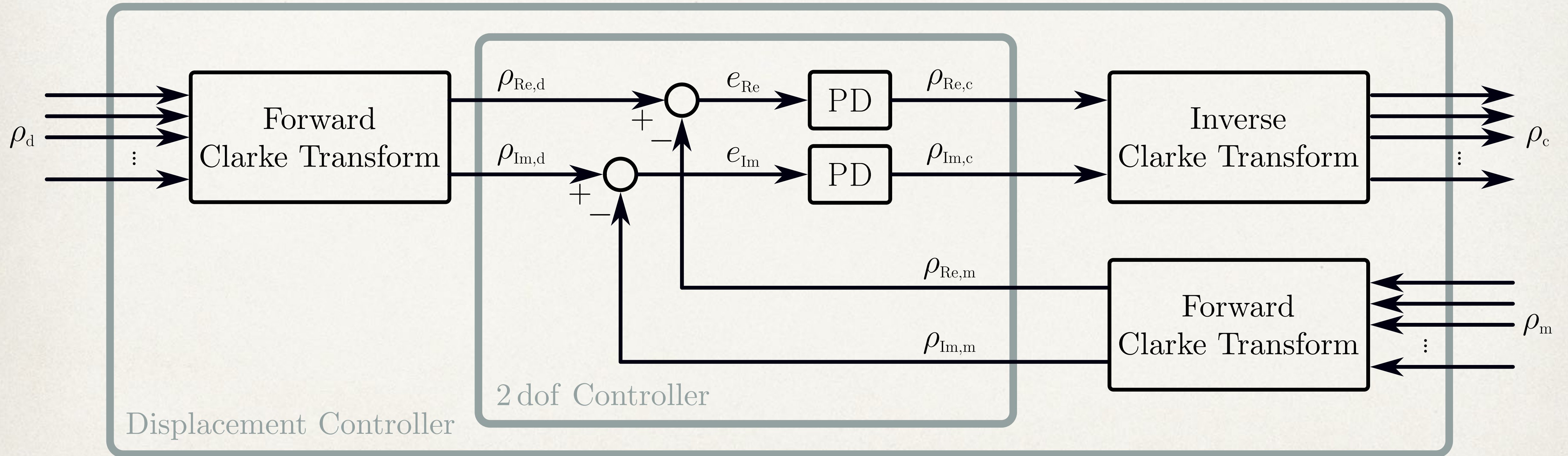




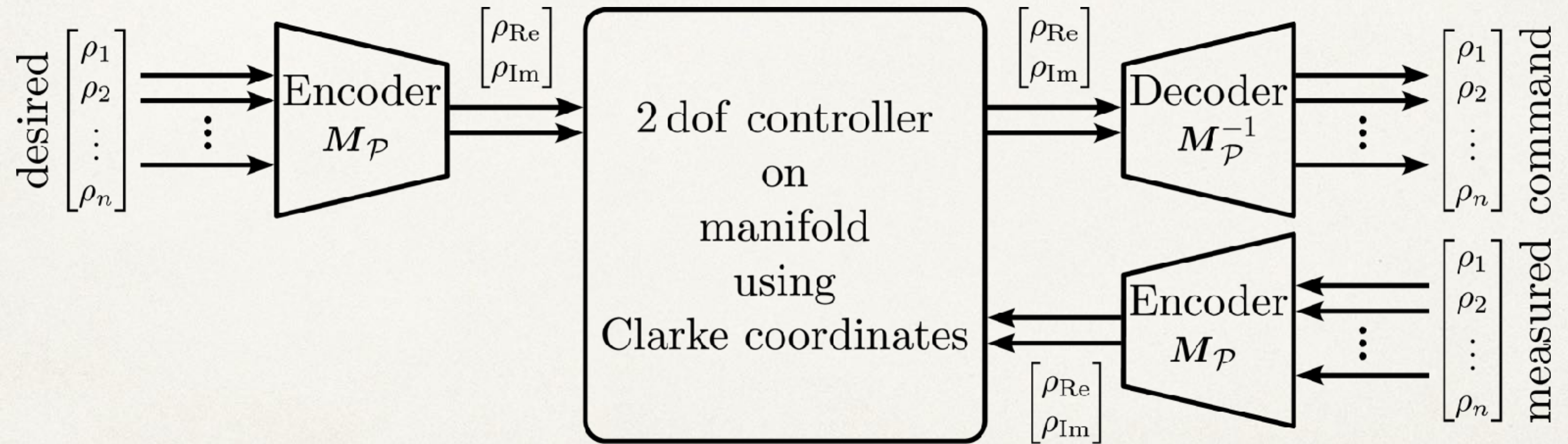
[Grassmann et al., arXiv (under review)] “Clarke Transform — A Fundamental Tool for Continuum Robotics”

[Grassmann & Burgner-Kahrs, arXiv (under review)] “Using Clarke Transform to Create a Framework on the Manifold”

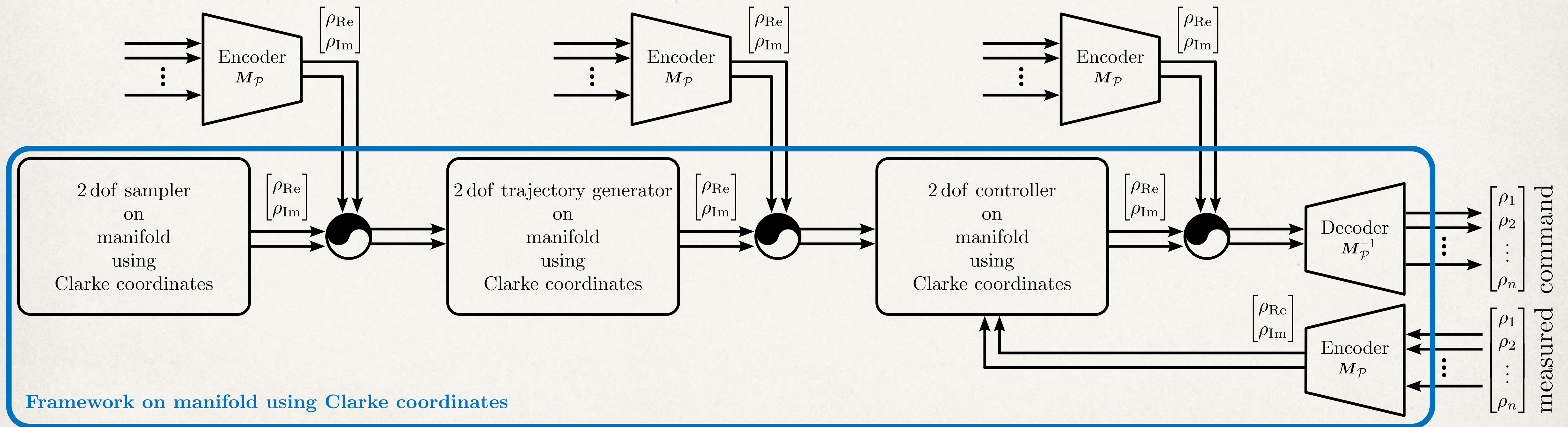
Simple Units



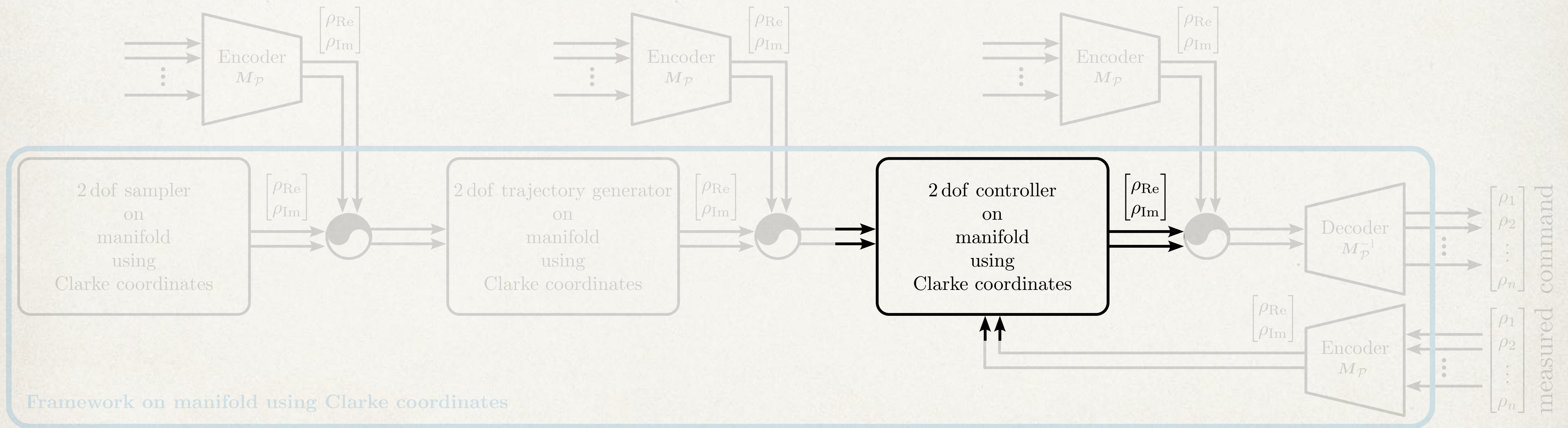
Simple Units



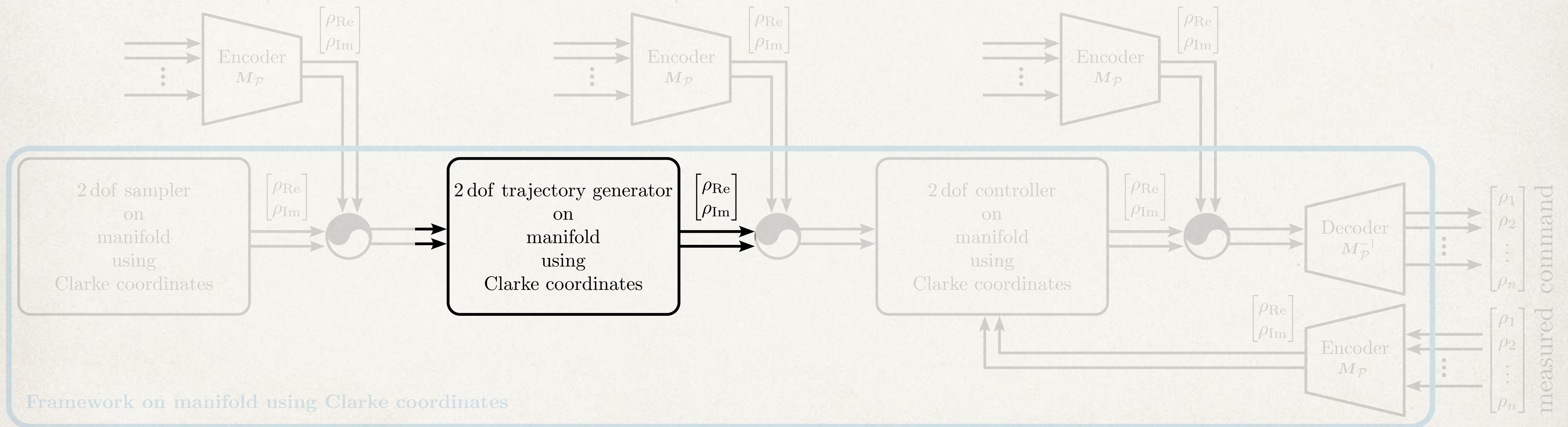
Concatenation of Units



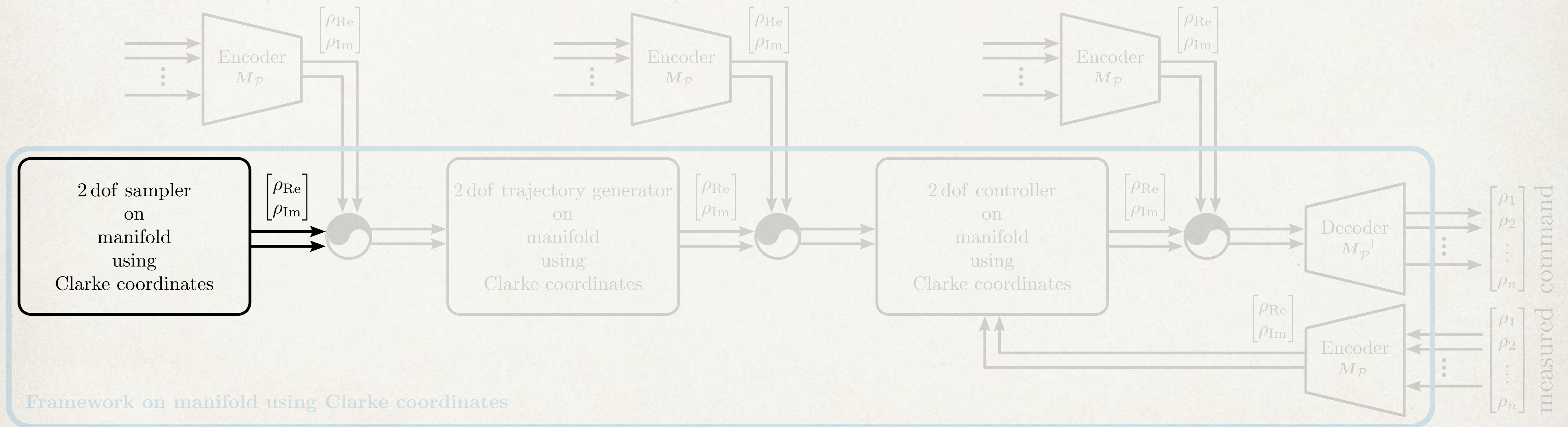
Concatenation of Units



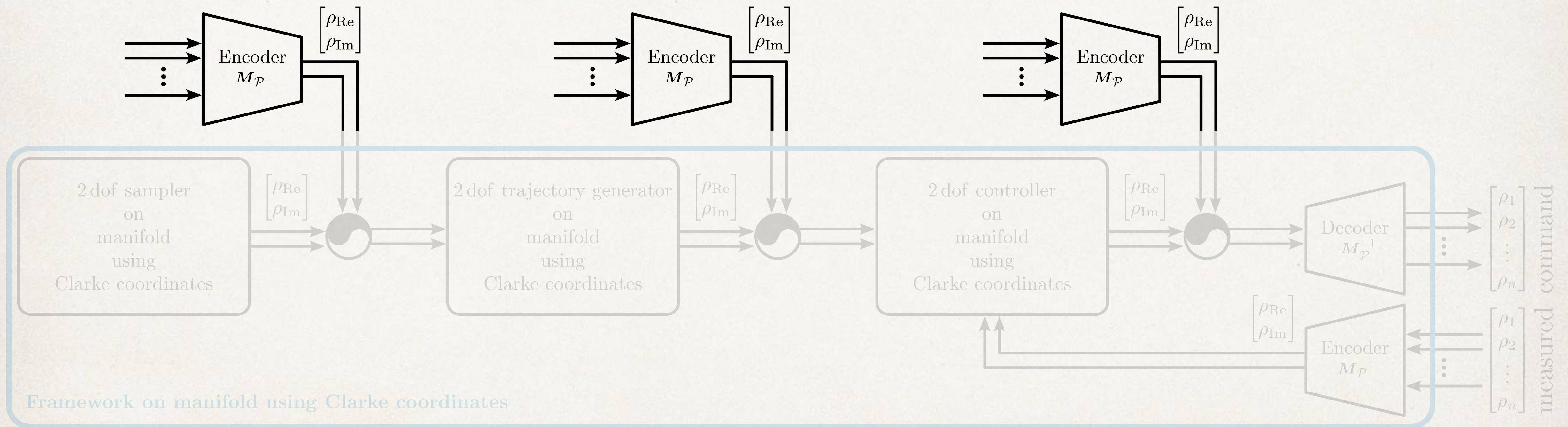
Concatenation of Units



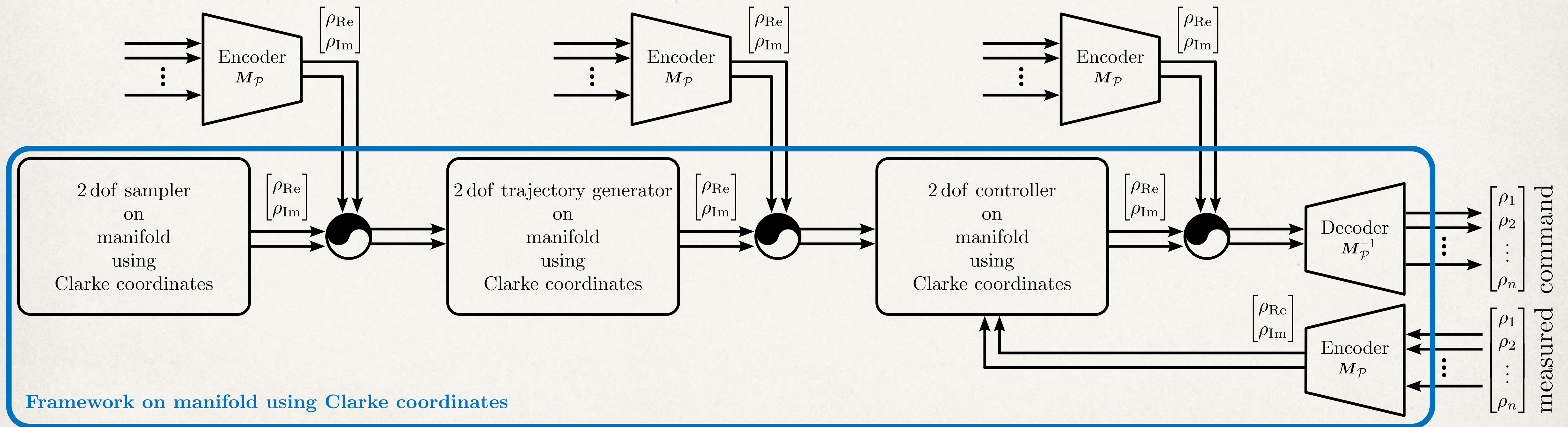
Concatenation of Units



Concatenation of Units



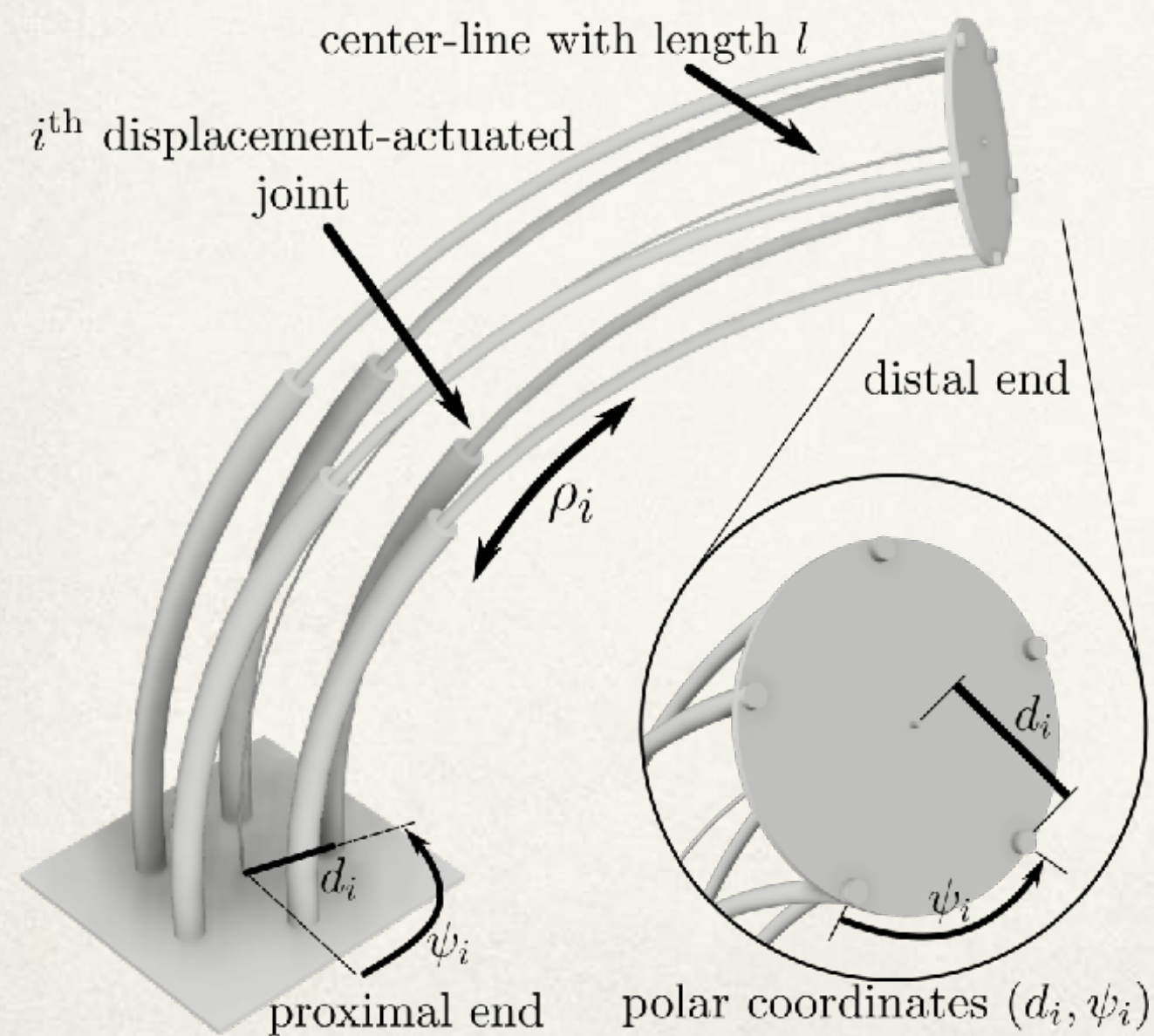
Concatenation of Units



Takeaway: Prelude of Benchmarking

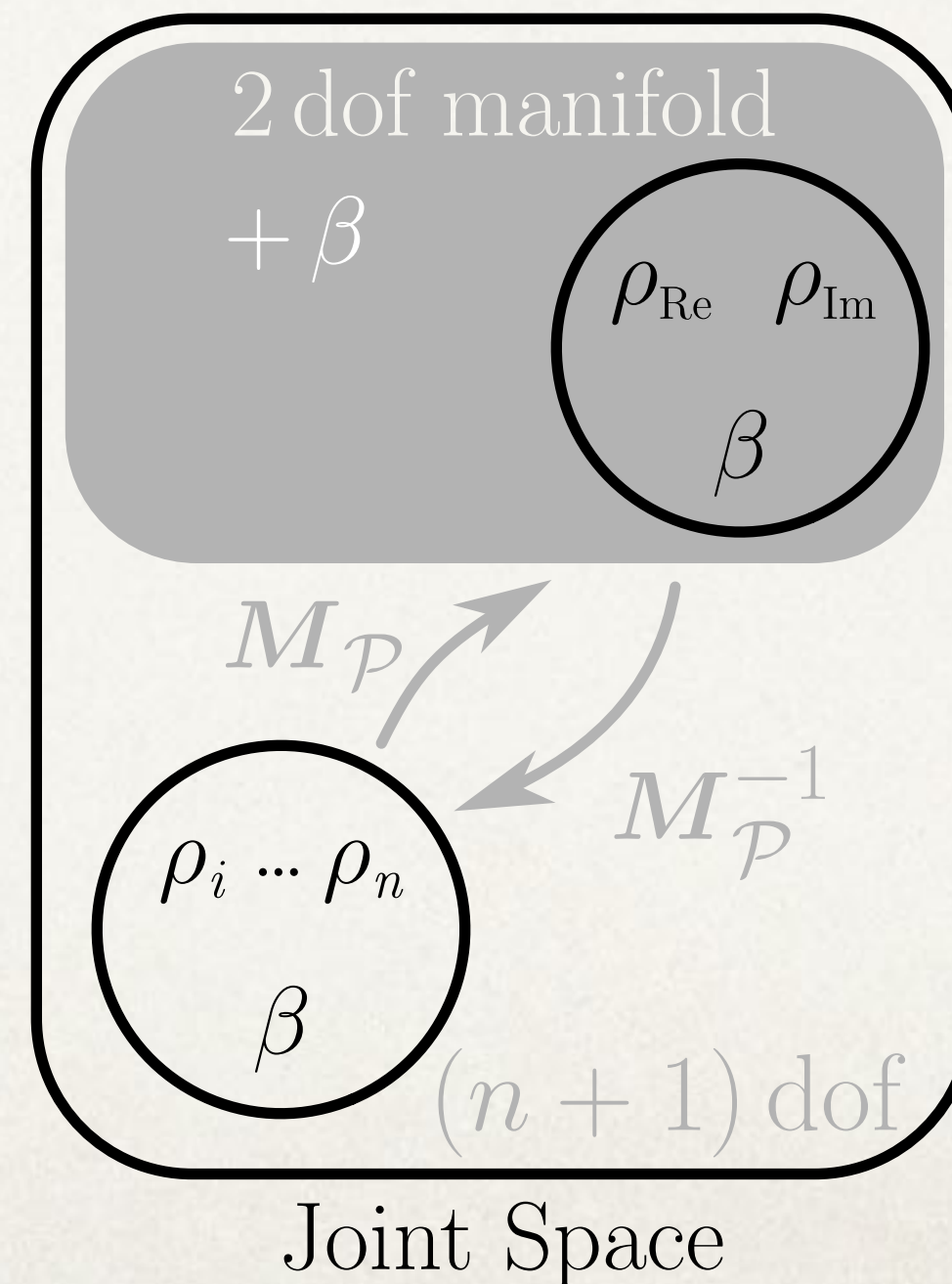
I.

right definition and abstraction



II.

manifolds and parameterization



III.

testable units

